A NOTE ON THE USE OF COEFFICIENT OF VARIATION IN ESTIMATING MEAN

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Let $\bar{x}$ and s^2 be the sample mean and the sample variance based on n sample observations drawn from a population with mean $\bar{X}$ and variance σ^2. In order to estimate $\bar{X}$, Searles[1] proposed an estimator

$$\hat{\bar{X}}_1 = \frac{n}{n+C}\bar{x}, \qquad \ldots(1)$$

where

$$C\left(=\frac{\sigma^2}{\bar{X}^2}\right)$$

is the square of coefficient of variation.

The mean-squared error of the estimator $\hat{\bar{X}}_1$ is smaller than the variance of the sample mean $\bar{x}$.

But, in practice, C is seldom known leaving the estimator (1) to be of no practical utility. In such a situation Srivastava[2] suggested to replace C in $\hat{\bar{X}}_1$ by its consistent estimator. One of the estimators considered by Srivastava[2] is

$$\hat{\bar{X}}_2 = \bar{x} - \frac{s^2}{n\bar{x}} \qquad \ldots(2)$$

In this note, we consider a slightly more general class of estimators for the mean as

$$\hat{\bar{X}} = \bar{x} + K\,\frac{s^2}{n\bar{x}}, \qquad \ldots(3)$$

where K is the characterising scalar to be chosen suitably.

It is obvious that (2) is a special case of (3) when $K=-1$. If $K=0$, we get the usual unbiased estimator.

The bias and the mean-squared error of $\hat{\bar{X}}$ are :

$$\text{Bias } (\hat{\bar{X}}) = \frac{K}{n} E\left(\frac{s^2}{\bar{x}}\right), \qquad \ldots(4)$$

$$\text{MSE}(\hat{\bar{X}}) = (1+2K)\frac{\sigma^2}{n} - 2K\frac{\bar{X}}{n} E\left(\frac{s^2}{\bar{x}}\right) + \frac{K^2}{n^2} E\left(\frac{s^2}{\bar{x}}\right)^2 \qquad \ldots(5)$$

Following the method of large sample approximations as discussed in Srivastava[2], we get the bias and the mean-squared error of $\hat{\bar{X}}$, to order $0(n^{-2})$,

$$\text{Bias } (\hat{\bar{X}}) = \frac{K}{n} C\bar{X}\left[1+\frac{1}{n}\left(C-\sqrt{\beta_1 C}\right)\right], \qquad \ldots(6)$$

$$\text{MSE } (\hat{\bar{X}}) = \frac{\sigma^2}{n}\left[1+\frac{K}{n}\left\{(K-2)C+2\sqrt{\beta_1 C}\right\}\right], \qquad \ldots(7)$$

where

$$\beta_1 \left(=\frac{\mu_3^2}{\sigma^6}\right)$$

is the coefficient of sknewness of the population.

From (7), the relative efficiency of $\hat{\bar{X}}$ with respect to $\bar{x}$ is

$$\text{REF}(\hat{\bar{X}},\bar{x}) = \frac{1}{\left[1+\frac{K}{n}\left\{(K-2)C+2\sqrt{\beta_1 C}\right\}\right]} \qquad \ldots(8)$$

Now, it follows that the proposed estimator $\hat{\bar{X}}$ is more efficient than $\bar{x}$ if we choose K such that it lies between 0 and

$$2\left[1-\sqrt{\frac{\beta_1}{C}}\right].$$

If we minimize (7) with respect to K, we find

$$K = 1-\sqrt{\frac{\beta_1}{C}} = K_{min} \text{ (say)} \qquad \ldots(9)$$

and, for this value of K, the mean-squared error, to order $0(n^{-2})$, is given by

$$\text{MSE}\left(\hat{\bar{X}}/K_{min}\right)=\frac{\sigma^2}{n}\left[1-\frac{C}{n}\left(1-\sqrt{\frac{\beta_1}{C}}\right)^2\right] \quad ...(10)$$

'In estimating the mean of symmetrical distributions ($\beta_1=0$) by estimators of the form $\hat{\bar{X}}=\bar{x}+K\frac{s^2}{n\bar{x}}$, the minimum mean-squared error is attained at $K=1$; therefore the estimator $\bar{X}_2$ considered by Srivastava[2] does not possess this property. Moreover, for $0<K<2$ the proposed estimator has mean-squared error smaller than the variance of the unbiased estimator $\bar{x}$.

REFERENCES

[1] Searles, D.T. *The Utilization of a Known Coefficient of Variation in Estimation Procedure* Journ. Amer. Stat. Assn., Vol. 59, 1964, pp. 1225-6.

[2] Srivastava, V.K. : *On the Use of Coefficient of Variation in Estimating Mean* Journ. Ind. Soc. Agri. Stat., Vol. XXVI No. 2. 1974, pp. 33-36.