A MODIFIED RATIO ESTIMATOR BASED ON THE COEFFICIENT OF VARIATION IN DOUBLE SAMPLING

D.M. KAWATHEKAR AND S.G. PRABHU AJGAONKAR

Marathwada University, Aurangabad

(Received : April, 1983)

SUMMARY

Utilising the value of coefficient of variation for an auxiliary variable, a modified ratio estimator for double sampling is proposed which is more efficient than the usual ratio estimator in double sampling and the simple mean estimator when ρ_{yx} lies between certain range.

INTRODUCTION

Sisodia and Dwivedi [2] noted that by utilising the value of C_x, coefficient of variation (C.V.) for an auxiliary variable, a modified ratio estimator is more efficient than both ratio and simple mean estimator when ρ_{yx} lies within a certain range. When information on population mean $\bar{X}_N$ is not available double sampling is used. The usual biased ratio estimator in double sampling is given by

$$\hat{Y}_{RD} = \frac{\bar{y}_n}{\bar{x}_n}\bar{x}_{n'}, \qquad \ldots(1.1)$$

and its bias and the mean square error are given by Sukhatme and Sukhatme [3].

In the present paper utilising the value of C_x, a modified ratio estimator in double sampling is proposed and its efficiency has been compared with the corresponding ratio estimator and the simple mean estimator. A cost fuction is also considered and the optimum values of n' and n, and the optimum variance of the proposed estimator are obtained. The efficiency of the proposed estimator has been illustrated with the example given by Jessen [1].

The proposed estimator is

$$\hat{\bar{Y}}_{MRD}=\frac{\bar{y}_n(\bar{x}_n'+C_x(}{(\bar{x}_n+C_x)} \quad ...(2.1)$$

It can be easily seen that to the first order of approximation

$$E(\hat{\bar{Y}}_{MRD})=\bar{Y}_N\left\{1+\left(\frac{1}{n}-\frac{1}{n'}\right)(C_x'^2-\rho_{yx}C_y\,C_x')\right\} \quad ...(2.2)$$

Where

$$C_x'=S_x/\bar{X}_N',\ \rho_{yx}=S_{yx}/S_xS_y,\ C_y=S_y/\bar{Y}_N,\ \bar{X}_N'=\bar{X}_N+C_x$$

With its relative bias given by

$$B^*=\left(\frac{1}{n}-\frac{1}{n'}\right)(C_x'^2-\rho_{yx}C_y\,C_x') \quad ...(2.3)$$

and MSE given by

$$M^*=\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{S_y^2+\frac{\bar{Y}_N^2}{\bar{X}_N'^2}S_x^2-2\frac{\bar{Y}_N}{\bar{X}_N'}S_{yx}\right\}+\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2 \quad ...(2.4)$$

If $n'=N$, the *MSE* of $\hat{\bar{Y}}_{MRD}$ to the first order of approximation reduces to *MSE* of modified ratio estimator proposed by Sisodia and Dwivedi (1981). When the information about C_x is not used we get $\bar{X}_N=\bar{X}_N'$, the *MSE* of proposed estimator reduces to the *MSE* of the usual ratio estimator in double sampling.

COMPARISION OF $\hat{\bar{Y}}_{MRD}$ WITH THE USUAL RATIO ESTIMATOR IN THE DOUBLE SAMPLING AND WITH THE SIMPLE MEAN ESTIMATOR :

The estimator $\hat{\bar{Y}}_{MRD}$ will be more efficient than $\hat{\bar{Y}}_{RD}$ if,

$$M>M^*$$

$$i.e.\quad R_N^2S_X^2-2R_N\,\rho_{yx}S_xS_y>\left(\frac{\bar{Y}_N}{\bar{X}_N'}\right)^2S_x^2-2\left(\frac{\bar{Y}_N}{\bar{X}_N'}\right)S_xS_y\,\rho_{yx}$$

$$i.e.\quad \rho_{yx}<\frac{1}{2}\frac{C_x}{C_y}\left(\frac{2\bar{X}_N+C_x}{\bar{X}_N+C_x}\right) \quad ...(3.1)$$

The estimator $\hat{\bar{Y}}_{MRD}$ will be more efficient than the simple mean estimator if

$$V(\hat{\bar{Y}}_n) > M^*$$

i.e. $$\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2 > \left(\frac{1}{n}-\frac{1}{n'}\right)\left\{S_y^2+\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)^2 S_x^2-2\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)\rho_{yx}S_xS_y\right\}$$

$$+\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2$$

i.e. $\rho_{yx} > \frac{1}{2}\, C'_x/C_y$ ...(3.2)

Combining (3.1) and (3.2) we find that the modified ratio estimator in double sampling is more efficient than the ratio estimator in double sampling and simple mean estimator if

$$\frac{1}{2}\,\frac{C_x}{C_y\left(1+\frac{C_x}{\bar{X}_N}\right)} < \rho_{yx} < \frac{1}{2}\,\frac{C_x}{C_y}\left(\frac{2\bar{X}_N+C_x}{\bar{X}_N+C_x}\right)$$

Cost Function

Consider the simple cost function

$$C = n'C' + nC'' \qquad \text{...(4.1)}$$

Where C' and C'' are the unit costs of observing an X_i [in phase *I*] and Y_i [in phase *II*] respectively and C is the total budget. It can be shown that

$$\text{Opt. Var. } (\hat{\bar{Y}}_{MRD}) = \frac{(\sqrt{C'S_1'^2}+\sqrt{C''}\,S_{11}'^2)^2}{C} \qquad \text{...(4.2)}$$

where

$$S_1'^2 = 2\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)S_{yx}-\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)^2 S_x^2$$

$$S_{11}'^2 = S_y^2-2\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)S_{yx}+\left(\frac{\bar{Y}_N}{\bar{X}'_N}\right)^2 S_x^2$$

N	C	C''	C'	$\bar{y}_n$	$\bar{x}_{n'}$	$\bar{x}_n$	C_x	C_y	C_{xy}
5000	100	1.000	0.10	80	22	20	.924	1.00	.832

$\rho_{xy} = 0.9$

Utilising the value of C.V. for auxiliary variable, we obtain the following optimum values of n' and n and the optimum variance

$$n'=409, n=59, \text{opt. Var. } (\hat{Y}_{MRD})=38.3$$

Whereas, without using the C.V. the optimum values of n' and n and optimum variance given by Jessen [1] are $n'=408$, $n=59$, opt Var. $(\hat{Y}_{RD})=42.7)$. Hence, the efficiency of the purposed estimator relative to the usual estimator is 111 per cent.

References

[1]. Jessen R.J (1978) : *Statistical survey techniques* John Wiley and Sons, New York.

[2] Sisodia B.V.S. and Dwivedi V.K. (1981) A modified ratio estimator using coefficient of variation of auxiliary variable. *Jour. of the Ind. Soc. of Agril. Stat.* Vol. XXXIII, No. 1, 13-18.

[3] Sukhatme P.V. and Sukhatme B.V. (1970) *Sampling theory of surveys with application* : Indian Society of Agricultural Statistics, New Delhi.