CONFOUNDING DIALLEL EXPERIMENTS—II

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1. Introduction

While presenting the analysis of some two-three-and four-class *PBIB* designs through latent roots and latent vectors of their *C*-matrices, Aggarwal ([1], [2], [3]) discussed the analysis of the confounding diallel experiments for the Methods (1), (3) and (4) of Griffing [5]. Some series of these PBIB designs which can be taken as useful confounded diallel experiments under different situations were also reported therein. This paper contains the analysis of the confounded diallel experiments known as the Method (2) of Griffing [5]. With u inbred lines, the $[u(u+1)/2]-1$ degrees of freedom for this method of Griffing, are partitioned into three orthogonal sets of $(u-1)$, 1 and $(u+1)(u-2)/2$ degrees of freedom, said to belong to general combining ability (*g.c.a.*), parents vs hybrids and specific combining ability (*s.c.a.*) effects, respectively. This partitioning is done by giving another characertization to the triangular association scheme of Bose and Shimamoto [4].

2. Definitions

We give another characertization of the triangular association scheme of Bose and Shimamoto [4] as following :

Definition 2.1. Let the $v=u(u+1)/2$ treatments be represented by an $u \times u$ square array

$$\begin{array}{cccc} 11 & 12 & \dots & 1u \\ 21 & 22 & \dots & 2u \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ u1 & u2 & \dots & uu \end{array} \qquad \dots(2.1)$$

The $u(u+1)/2$ treatments of this array, can be denoted by ij $(i, j=1, 2, ..., u;\ ij=ji)$. For a treatment ij $(i \neq j)$

(1) first associates are the treatments occuring in the same row and column in which the treatment ij $(i \neq j)$ occurs; and

(2) all other treatments are second associates.

For a treatment ii

(1) first associates are the treatments occuring in the same row or column in which the treatment ii occurs and the treatments occuring in the diagonal positions of the array (2.1) from the left top to the right bottom; and

(2) all other treatments and second associates.

Definition 2.2. A PBIB design with the triangular association scheme, is called a triangular design.

Let N be the incidence matrix of a connected triangular design with the parameters $v=u\ (u+1)/2,\ b,\ r,\ k,\ \lambda_1,\ \lambda_2$. Then the latent roots θ_i of NN' with their multiplicities α_i $(i=0, 1, 2)$ will be

$$\theta_0=rk,\ \alpha_0=1;\ \theta_1=r+(u-3)\ \lambda_1-(u-2)\ \lambda_2,\ \alpha_1=u;$$

$$\theta_2=r-2\ \lambda_1+\lambda_2,\ \alpha_2=(u+1)\ (u-2)/2, \qquad ...(2.2)$$

The latent roots ϕ_i of the C-matrix of this triangular design with their multiplicies α_i, are given by

$$\phi_i=r-\theta_i/k$$

$$(i=0, 1, 2). \qquad ...(2.3)$$

Let there be u inbred lines. Let us consider $u\ (u-1)/2\ F_1$'s and u parents. Let these $u\ (u+1)/2$ crosses be denoted by the treatments ij $(i, j=1, 2, ..., u;\ ij=ji)$ of a connected triangular design with the parameters $v=u\ (u+1)/2,\ b,\ r,\ k,\ \lambda_1,\ \lambda_2$. Then the $(u(u+1)/2)-1$ degrees of freedom can be partitioned into three orthogonal sets of $(u-1),\ 1,\ (u+1)\ (u-2)/2$ degrees of freedom said to belong to *g.c.a.*, parents vs hydrids and *s.c.a.*, effects, respectively, as discussed in detail in the next section.

3. Analysis

(a) *Fixed effects model*

Let Y_{ijl}, the observed value of the plot in the l"th block to which the ijth treatment is allotted, be given by

$$Y_{ijl}=m+t_{ij}+\beta_l+e_{ijl},$$
$$i, j=1, 2, \ldots, u;$$
$$ij=ji;$$
$$l=1, 2, \ldots, b \qquad (3.1)$$

m is called the general mean, t_{ij} is called the ijth treatment effect and β_l is called lth block effect, Let m, t_{ij}'s and β_l's be the fixed-effects. Let e_{ijl}'s be normally and independently distributed with expectations 0 and variances σ^2 Let $\sum\sum_{i\leq j} t_{ij}=0$. The equation (3.1) with these assumptions is called a fixed-effects model I (see Scheffe [7], p. 6).

Let $Y\ldots$denote the total of all the observations in the experiment. Let T_{ij} be the total of all the observations for the ijth treatment. Let B^{ij} be the total of all the blocks in which the ijth treatment occurs. Then Q_{ij}, the adjusted treatment total is given by $T_{ij}-(1/k)\,B^{ij}$. Let $\hat{t}_{ij}$ be the least square estimate or t_{ij} the true effect or the ijth treatment.

Let

$$\underline{p}=(p_{11}, p_{12}, \ldots, p_{1u}, p_{22}, \ldots, p_{2u}, \ldots, p_{(u-1)u}, p_{uu})'.$$

$$p_i=\sum_j p_{ij}, p_j=\sum_i p_{ij}; D^p=\sum_i p_{ii}; \quad \ldots(3.2)$$

$$\triangle^p=\sum\sum_{i<j} p_{ij};\ p=t, Q, \hat{t}.$$

Let

$$t_{ij}=g_i+g_j+s_{ij} \qquad \ldots (3.3)$$

where g_i and g_j are the common genic contributions of the ith maternal line and the jth paternal line. g_i is called the general combining ability (*g.c.a.*) effect of the ith line; s_{ij} is the interaction between the genic contributions of the ith maternal line and jth paternal line, s_{ij} is called the specific combining ability (*s.c.a.*) effect due to the ijth cross.

Let us further assume

$$\sum_j s_{ij}=0, \quad \text{for all } i; \qquad \text{.. (3.4)}$$

$$\bar{g}=\left(\sum g_i\right)/u.$$

Then it can easily be seen that

$$g_i=(t_i/u)+(D^t/2u^2);\ s_{ij}=t_{ij}-g_i-g_j. \qquad \text{...(3.5)}$$

The latent vectors $\underline{x}_i$ ($i=2, 3, \ldots, u$) and $\underline{y}$ corresponding to the latent root ϕ_1 of the C-matrix, form the contrasts for the *g.c.a.* effects and parents vs hybrids,
respectively and the latent vectors $\underline{z_i}$ ($i=1, 2, \ldots, (u+1)\ (u-2)/2$) corresponding to the root ϕ_2 of the C-matrix, from the contrasts belonging to the *s.c.a.* effects. These three sets of contrasts are

$$\underline{t}'\underline{x}_i=\left(\sum_{j=1}^{i-1} t_{j.}-(i-1)\ t_{i.}\right) \div [(i-1)\ i\ (u-1)]^{\frac{1}{2}}$$

$$i=2, 3, \ldots, u;$$

$$\underline{t}'\underline{y}= [(u-1)\ D^t-2\ \Delta^t]\div[(u\ (u-1)\ (u+1)]^{\frac{1}{2}};$$

$$\underline{t}'\underline{z}_i(i=1, 2, \ldots, (u+1)\ u-2)/2) \qquad \text{...(3.6)}$$

where the vectors $\underline{z_1}$'s are orthonormal and orthogonal to the vectors $\underline{x}_1$'s and $\underline{y}$. It may be noted that the $(n-1)$ *g.c.a.* orthogonal contrasts will be g_1-g_2, $g_1+g_2-2g_3, \ldots, g_1+g_2+\ldots+g_{(u-1)}-(u-1)\ g_u$ which will be the same as given by $\underline{t}'\ \underline{x}_i$ ($i=2, 3, \ldots, u$) in (3.6) and $\underline{t}'\ \underline{y}$ represents parents vs hybrids contrast. The remaining $(u+1)\ (u-2)/2$ orthogonal contrasts are said to represent *s.c.a.* contrasts.

It may further be noted that the set of assumptions given in (3.4) is different from the set of assumptions (*i*) $\sum_i g_i=0$, and (*ii*) $\sum s_{ij}+s_{ii}=0$ given by Griffing ([5] p. 473). The (*ii*) set of assumptions, seems to be unrealistic and has to be assumed by Griffing to solve the normal equations. We do not assuming $\sum_i g_i=0$ but as it can be seen that we shall test the hypothesis $\sum_i g_i=0$. To solve the normal equations, we have assumed $\sum\sum_{i\leqq j} t_{ij}=0$. It may be noted that $\underline{t}'\,\underline{y}=0$ implies $D^t=0$ which implies

$$\sum_i g_i=0 \text{ or } \sum_i s_{ii}=0.$$

The parents vs hybrids 1 degr ee of freedom under the set of assumtions (3.4), can be used to test the significance of $\sum_i g_i$ or D^t or $\sum_i s_{it}$.

We shall, now, present the analysis. Let

$$A_1=\sum_{i=2}^{u} \underline{x}_i\,\underline{x}'_i,\ A_2=\underline{y}\,\underline{y}',\ A_3=\sum_{i=2} \underline{z}_i\,\underline{z}_i'. \qquad \text{...(3.7)}$$

Then a solution of the reduced normal equations will be given by

$$\hat{t}_{ij}=(Q_{ij}/\phi_2)+((1/\phi_1)-(1/\phi_2))\,(Q_{i\cdot}+Q_j)/(u-1),$$
$$i,j=1,2,\ldots,u;\ i<j; \qquad \text{...(3.8)}$$
$$\hat{t}_{ii}=(Q_{ii}/\phi_2)+((1/\phi_1)-(1/\phi_2))(Q_{i\cdot}+D^Q)/(u-1),$$
$$i=1,2,\ldots,u.$$

Following Aggarwal [1], the sum of squares pertaining to the *g.c.a.*, parents vs hybrids and *s.c.a.* degrees of freedom will be given as in the following theorem:

Theorem 3.1. In a connected triangular design, the sum of squares due to *g.c.a.*, effects parents vs hybrids and *s.c.a.* effects eliminating block effects, are $(1/\phi_1)\ \underline{Q}'\ A_1\ \underline{Q}$, $(1/\phi_1)\ \underline{Q}'\ A_2\ \underline{Q}$ and $(1/\phi_2)$ $\underline{Q}_3'\ A\ \underline{Q}$, respectively.

The Anova table giving the sum of squares ($S.S.$) due to the various effects and their expected mean squares is given in Table 3.1.

The equality of g_1's is tested by comparing M_g with the error mean square M_e. The F-ratio M_{ph}/M_e provides a test for the significance of parents vs hybrids contrast or

$$\Sigma\, s_{ii} \text{ or } \Sigma\, g_i \text{ or } \Sigma\, t_{ii}.$$

The signification of s_{ij}'s can be tested approximately as suggested in scheffe ([7], p. 247—48).

The least square estimates of the various parameters are given as follows :

$$\hat{g}_i=(Q_i/u\ \phi_1)+(D^Q\ /2u^2\ \phi_1),\ \hat{s}_{ij}=\hat{t}_{ij}-\hat{g}_i-\hat{g}_j \qquad \ldots(3.9)$$

and

$$V(\hat{g}_i-\hat{g})=2\ (u-1)\ \sigma^2/u^2\ \phi_1,\ i\neq j. \qquad \ldots(3.10)$$

The variances of elementary contrasts of s_{ij}'s are slightly complicated in form and can be found out from the well known result

$$V(\underline{l}'\hat{t})=\sigma^2\ \underline{l}'\ \bar{C}\ \underline{l}\ =\sigma^2\ \underline{l}'\ ((1/\phi_1)\ (A_1+A_2)+(1/\phi_2)\ A_3)\ \underline{l} \qquad \ldots(3.11)$$

where $\underline{l}'\ \underline{t}$ is some estimable contrast. It can be seen that the $S.S.$ due to the three types of contrasts given by (3.6), will be

$$\phi_1\ (\underline{x}'_i\ \hat{\underline{t}})^2\ , \qquad i=2, 3, \ldots, u$$

$$\phi_1\ (\underline{y}'\ \hat{\underline{t}})^2\ ,$$

$$\phi_2\ (\underline{z}'_i\ \hat{\underline{t}})^2\ , \qquad i=1, 2, \ldots, (u+1)\ (u-2)/2. \qquad \ldots(3.12)$$

The usual test procedure is followed to test the significance of these contrasts. Further, it may be noted that all the elementary contrasts pertaining to *g.c.a.* effects are estimable with equal precision, whereas elementary contrasts pertaining to *s.c.a.* effects, are not

TABLE 3.1.
Anova Table

Source	*d.f.*	*S.S.*	*M.S.*	*E(M.S.) Fixed-effects model*	*E(M.S.) mixed-effects modle*
Blocks ignoring treatments	$(b-1)$	$[(\Sigma B_l^2)/k]-Y^2\ldots/rv$	—	—	—
Treatments liminating blocks					
g.c.a.	$(u-1)$	$[\Sigma_i Q_{i.}^2-((D^Q)^2/u)]/(u-1)\,\Phi_1$	M_g	$\sigma^2+u^2\,\Phi_1\,\Sigma(g_i-\bar{g})^2/(u-1)^2$	$\sigma^2+u^2\,\Phi_1\,\sigma_g^2/(u-1)$ $+2(u+1)\,[2(u-1)$ $\Phi_1/(u+1)^2+\Phi_2^2/u^2\,\Phi_1]\,\sigma_s^2$
Parents vs Hybrids	1	$(u+1)\,(D^Q)^2/u\,(u-1)\,\Phi_1$	M_{ph}	$\sigma^2+u\Phi_1\,(D^s)^2/(u+1)\,(u-1)$	$\sigma^2+2(u+1)\,[2(u-1)$ $\Phi_1/(u+1)^2+\Phi_2^2/u^2\,\Phi_1]\,\sigma_s^2$
s.c.a.	$(u+1)$ $(u-2)/2$	$\sum\sum_{i\leq j} Q_{ij}^2$ $-[\,_i\,Q_{i.}^2+(D^Q)^2]/(u-1)]/\Phi_2$	M_s	$\sigma^2+2\Phi_2\,(g_1-\bar{g})^2/(u-1)\,(u+1)$ $+2\Phi_2\sum\sum_{i\leq j} s_{ij}^2/(u-2)$ $(u+1)+n^*\,(D^s)^2+4\,\Phi_2$ $\Sigma\,g_i\,s_{ii}/(u+1)\,(u-2)$	$\sigma^2+2\Phi_2\,\sigma_g^2/(u+1)$ $+\Phi_2\,\sigma_s^2$
Error	$v(r-1)$ $-b+1$	By substraction	M_e	σ^2	σ^2
Total	$vr-1$	$\Sigma\Sigma\Sigma\,Y_{ijl}^2-Y^2\ldots/rv$	—	—	—

n^* in $E(M.S.)$ for *s.c.a.* is a function of u, λ_1, λ_2, k only.

estimable with equal precision. The relative loss of information on each of the partially confounded degree of freedom can be worked out as suggested by Aggarwal [2].

For the series of triangular designs (See Raghavarao [6], p. 154) with the parameters

$$v=u\,(u+1)/2,\ b=u\,(u-2),\ r=(u-2),\ k=(u+1)/2,$$
$$\lambda_1=0,\ \lambda_2=1 \qquad \ldots(3.13)$$

the $(u-1)$ degrees of freedom pertaining to *g.c.a.* effects and one degree of freedom pertaining to parents vs hybrids is not confounded whereas the relative loss of information on each of the $(u+1)(u-2)/2$ partially confounded degrees of freedom pertaining to *s.c.a.* effects is $2\,(u-1)/(u-2)\,(u+1)$. For the series of triangular designs (See Raghavarao [6], p. 152) with the parameters

$$v=u\,(u+1)/2,\ b=(u+1),\ r=2,\ k=u,\ \lambda_1=1,\ \lambda_2=0 \qquad \ldots(3.14)$$

the relative loss of information on the partially confouned $(u-1)$ degree of freedom pertaining to *g.c.a.* effects and the one degree of freedom pertaining to parents vs hybrids is $(u-1)/2u$. The *s.c.a.* effects are left unconfounded.

(*b*) *Mixed effects model*

Let us again consider the relations given in (3.1) and (3.3). Let m, β_l's be taken as fixed effects. Let g_i's, s_{ij}'s and e_{ijl}'s be normally and independently distributed with expectations 0 and variances σ_g^2, σ_s^2 and σ^2, respectively. Further let these random variables be pairwise uncorrelated. That is cov $(g_i, s_{ij})=0$, cov $(g_i, e_{ij}\,l)=0$ and cov $(s_{ij}, e_{ij}\,l)=0$, for all i, j, l. The observational set up given in (3.1) and (3.3) with these assumptions, is called a mixed-effects model (See Scheffe [7], p. 6). The expectations of the various mean square under a mixed-effects model are also given in the table 3.1. For testing the significance of σ_g^2, the F-ratio M_g/M_{ph} is used and for testing the significance of σ_s^2 the ratio M_{ph}/M_e is used and usual test procedure is followed. The estimates of σ_g^2, σ_s^2 and σ^2 can, easily, be worked out from the Anova Table 3.1.

4. Illustration

Let us consider a triangular design with the parameters

$$v=6,\ b=4,\ r=2,\ k=3,\ \lambda_1=1,\ \lambda_2=0$$

and with the triangular association scheme

$$\begin{matrix} 11 & 12 & 13 \\ 12 & 22 & 23 \\ 13 & 23 & 33 \end{matrix} \quad \text{...(3.15)}$$

Let us assume an intrablock model. Let the 4 blocks of this triangular design (yields given within brackets) be as following :

(11(7), 22(10), 33(11)), (11(9), 12(14), 13(16)),
(22(11), 12(13), 23(17)), (33(13), 13(18), 23(20)). ...(3.16)

Then the matrix (Q_{ij}) will be

$$\begin{matrix} -6.33 & 0.33 & 4.00 \\ 0.33 & -2.00 & 6.33 \\ 4.00 & 6.33 & -2.33 \end{matrix} \quad \text{...(3.17)}$$

The anova is given Table 3.2.

TABLE 3.2

Anova Table

Source	*d.f.*	*S.S.*	*M.S.*	*F-ratio*
Blocks ignoring treatments	3	88.9200	—	—
g.c.a.	2	19.4389	9.7195	26.18*
Parents vs hybrids	1	56.8178	56.8178	158.79**
s.c.a.	2	2.0000	1.0000	2.79
Error	3	1.0733	0.3576	
Total	11	168.2500		

From the anova Table 3.2. it may be observed that the *g.c.a.* effects are significantly different at 5 *p.c.* level of significance and parents vs hybrids contrast is also significant at 1 *p.c.* level of significance. The least square estimates of g_1, g_2 and g_3 are -0.94, 0.64, and 1.56, respectively. *C.D.* for the *g.c.a.* effects at 5 percent level of significance is 1.08.

SUMMARY

In this paper, analysis of the confounded diallel experiments for the Method (2) Griffing [5] is presented. With u imbred lines, the $(u(u+1)/2)-1$ degrees of freedom for this method of Griffing, are partitioned into three orthogonal sets of $(u-1)$, 1 and $(u+1)(u-2)/2$ degrees of freedom, said to belong to general combining ability (*g.c.a.*) parents vs hybrids and specific combining ability (*s.c a.*) effects, respectively.

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