ON SOME GENERAL PROPERTIES OF THE 'MINIMUM RECOMMENDED RATE' IN FERTILIZER APPLICATION

By P. N. Saxena

Indian Agricultural Research Institute, New Delhi

Introduction

The concept of the 'optimum rate', defined as the rate of fertilizer giving the maximum net profit *per unit area* from its use, and representing the upper limit in fertilizer recommendations, is now fairly well known. A large number of studies dealing with the determination of these rates for one or more factor inputs and using various forms for the fertilizer-yield response functions have been made. Mention may be made in this connection of the work of Spillman (1933), Crowther and Yates (1941), Sukhatme (1941), Panse (1945), Paschal (1953), Heady *et al.* (1955) and Ibach (1956).

Farmers with adequate capital or credit can profitably fertilize upto the extent indicated by the optimum rates. Limited capital or fertilizer resources may, however, make it impossible to apply rates this high. Under these circumstances the cost of application of fertilizer represents a significant part of the total fertilizer cost. As pointed out by Mitscherlich (1909), very low rates now become unprofitable in this situation, and a certain *minimum* rate of fertilizer must be applied before the break-even point is reached. It is only recently that Pesek and Heady (1958) have explicitly defined this lower limit for agronomic fertilizer recommendations as the rate giving the maximum return *per dollar* (*or rupee*) invested in fertilization.

The object of the present paper is to elucidate certain important general properties of this concept. In addition, the situation, where the harvesting and threshing cost of extra produce from the use of fertilizer needs to be accounted for, has also been investigated here and the results extended to the case of two or more input variables.

2. Graphical Representation

In order to get a clear picture of the concepts involved it is useful to approach the problem graphically. We shall assume that the response curve is *concave* to the X- or fertilizer rate-axis throughout the relevant range of response. It is also assumed that the cost of application of fertilizer, determined on *area* basis, is a *constant*, so that the total cost of fertilizer can be expressed in a *linear* form in x.

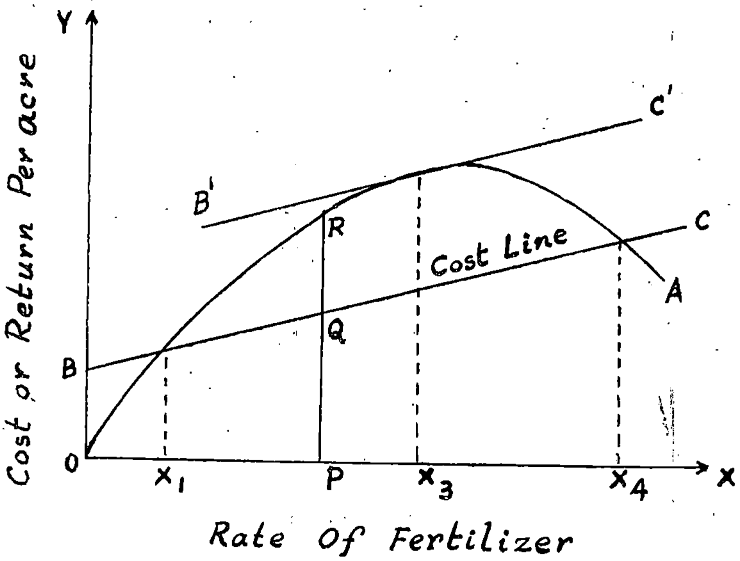

FIG. 1

Let OA (Fig. 1) represent the crop response curve to fertilizer, expressed in *value* terms, so that ordinates up to the curve represent the gross returns per acre by fertilization. Let OB be the fixed cost of application of fertilizer per acre, so that the line BC represents the total (*i.e.*, fixed + variable) rupee cost of fertilization per acre. Clearly, the vertical distances between the straight line BC and the response curve OA represent the net returns (+ or −) per acre. It is now evident that fertilization with rates below X_1 or above X_4 leads to a net loss, and that the net return is a maximum at a point where a line $B'C'$, parallel to BC, is tangential to the response curve. This leads to the determination of the 'optimum' rate of fertilization X_3.

Consider now a situation where it is not practicable to apply the optimum rate of fertilizer. Plotting the average net returns per unit of money invested in fertilization as given by the ratios QR/PQ, we then arrive at a diagram of the form given in Fig. 2.

It is obvious from Fig. 2 that the average net return is negative for points below X_1 or above X_4 and is positive for rates between these two extreme values. This average net return per rupee invested in fertilization rises to a maximum at a point X_2 and then declines.

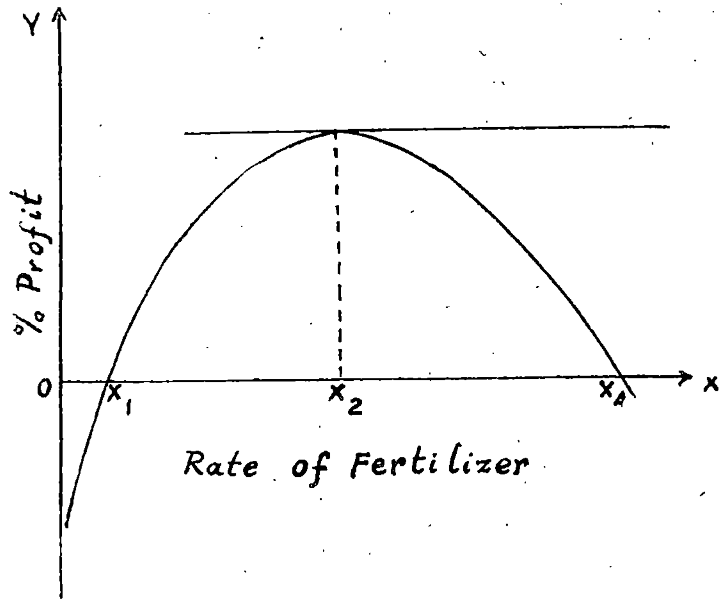

FIG. 2

The rate X_2 represents the most efficient use of fertilizer resources per rupee invested, and defines Pesek and Heady's 'Minimum Recommended Rate'. Below this point both the profits per acre and the percentage profit decrease.

It is also evident from Fig. 1 above that the rate X_2 is less than X_3, since the % profit cannot increase beyond X_3 where the net returns decline with increasing total costs of fertilization.

3. ALGEBRAIC FORMULATION

We shall now formulate the problem algebraically. Let

$$Y_1 = f(x) \tag{1}$$

represent the response function, positive and concave to the X-axi over the range under consideration, in units of crop yield (mds. per acre), for a rate 'x' units of fertilizer applied per acre. Let, further, in terms of rupees,

p = Price pre unit of crop yield,

q = Cost per unit of fertilizer,

a = Cost of application per acre of fertilizer.

Hence the total fertilizer cost per acre is given by $(a + q\,x)$ rupees. In terms of crop yield, this cost of fertilization per acre may then be represented by

$$Y_2 = \frac{a}{p} + \frac{q}{p}\,x. \tag{2}$$

We thus have,

$$\text{Investment in Fertilization (Rs. per acre)} = pY_2 = a + qx \tag{3}$$

$$\text{Net profit (Rs. per acre)} = p\,(Y_1 - Y_2) = pf(x) - (a+qx). \tag{4}$$

The 'optimum' rate maximises the net profit per acre given by $p\,(Y_1 - Y_2)$ in (4). The 'minimum recommended rate' (M.R.R.) maximises the net return per rupee invested in fertilization, *i.e.*, the ratio $p\,(Y_1 - Y_2)/pY_2$. In other words, the M.R.R. is determined by maximising the expression:

$$\frac{Y_1}{Y_2} = \frac{f(x)}{\frac{a}{p} + \frac{q}{p}\,x} = \frac{pf(x)}{a + qx} \tag{5}$$

4. Some General Properties of the Minimum Recommended Rate of Fertilization

We are now in a position to prove the following theorems:—

Theorem 1.—The optimum rate is independent of 'a' the cost of application of fertilizer per acre. The M.R.R. is independent of 'p' the price of produce per unit, and depends only on the ratio a/q, in addition to the constants of the response function.

The solutions to the 'optimum' and 'minimum' rates are obtained by equating the differential coefficients of the functions in (4) and (5) to zero. Representing the differential coefficient of $f(x)$ by $f'(x)$ we thus arrive at the following equations for determining the 'optimum' and 'minimum' rates respectively:—

$$\text{For optimum rate:} \quad f'(x) = \frac{q}{p}. \tag{6}$$

$$\text{For minimum rate:} \quad \frac{f(x)}{f'(x)} = x + \frac{a}{q}. \tag{7}$$

The statements in the theorem are now obvious. The results, although quite easily derived, apply to any form of the response function. They also throw into sharp relief certain errors in the values of the M.R.R., using a quadratic response function, in Table I of Pesek and Heady's (1958) paper (*vide* the values 17·0, 17·1 and 17·2 lb. P_2O_5 per acre of the minimum recommended rate of fertilization for three alternative situations in each of which $a/q = 5$).

Theorem 2.—Under limited capital conditions the *total* profit is maximised by using the M.R.R. of fertilizer.

Let the total investment in fertilization be fixed at I rupees. Using 'x' units of fertilizer per acre the investment per acre in fertilization is then $(a + qx)$ rupees as in (3). Thus, with this limited capital,

$$\text{No. of acres which can be fertilized} = \frac{I}{(a + qx)}. \tag{8}$$

Using (8) in conjunction with the net profit per acre given in (4), we then have:

$$\begin{aligned}\text{Total profit (Rs.)} &= \text{Net profit per acre} \times \text{No. of acres fertilized} \\ &= p\,(Y_1 - Y_2) \times \frac{I}{(a + qx)} \\ &= p\,(Y_1 - Y_2) \times \frac{I}{pY_2} \\ &= \left(\frac{Y_1}{Y_2} - 1\right) I. \end{aligned} \tag{9}$$

Since I is fixed, it follows from (5) and (9) that the total profit in this situation is maximised by using the M.R.R. of fertilizer. An illustration of this result is provided by Table II of Pesek and Heady (1958), where the total profit by fertilizing more or less acres within a given amount available for fertilization has been worked out.

Theorem 3.—The M.R.R. of fertilizer increases (or decreases) with a/q.

Let us write equation (7) determining the M.R.R. in the following form:

$$\frac{a}{q} = \frac{f(x)}{f'(x)} - x = u(x), \text{ say.} \tag{10}$$

Hence, on differentiating (10), we obtain,

$$\frac{du}{dx} = -\frac{f(x)f''(x)}{[f'(x)]^2} \tag{11}$$

where

$$f'(x) = \frac{d}{dx} f(x)$$

and

$$f''(x) = \frac{d^2}{dx^2} f(x).$$

Now the response function $f(x)$ is positive for the values of x under consideration. It has also been assumed that $f(x)$ is *concave* to the X-axis throughout the range, so that by the definition of concavity,

$$f(x).\ f''(x) = -\text{ ve},$$

i.e., for $f(x) > 0$, we have,

$$f''(x) = -\text{ ve}.$$

Hence, we have

$$\frac{du}{dx} = +\text{ ve}, \quad X_1 \leqslant x \leqslant X_4. \tag{12}$$

It follows from (12) that $u(x)$ and x increase (or decrease) together. In other words, the M.R.R. as a solution of (10) increases (or decreases) with a/q. We thus conclude that for a fixed cost 'q' of fertilizer, the M.R.R. increases with the cost of application—since it takes more in this case to reach the break-even point (X_1 in Fig. 1). Conversely, for a fixed cost of application 'a' per acre, the M.R.R. decreases with increasing cost of fertilizer per acre.

5. Harvesting and Threshing Costs of Extra Yields

We shall now somewhat modify the formulation of the problem given in Section 3 so as to take into account the additional cost incurred in harvesting and threshing the extra produce obtained with the use of fertilizer. The effect of this change on the 'optimum' and 'minimum' rates of fertilization will then be investigated.

Let, the cost of harvesting and threshing additional produce

$$= h \text{ Rupees per md.}$$

Then, with the other symbols as before, the following relations hold in terms of units of crop yield, *i.e.*, maunds/acre.

Response function:

$$Y_1 = f(x). \tag{13}$$

Cost function:

$$Y_2 = \frac{a}{p} + \frac{q}{p} x + \frac{h}{p} \cdot f(x). \tag{14}$$

Thus,

$$\text{Investment in fertilization (Rs./acre)} = pY_2 = a + qx + hf(x). \tag{15}$$

Hence,

$$\text{Net profit (Rs./acre)} = p(Y_1 - Y_2) = (p - h)\, f(x) - (a + qx). \tag{16}$$

The optimum rate of fertilization obtained by maximising the net profit per acre given by $p(Y_1 - Y_2)$ in (16) is then found as the solution of:

$$f'(x) = \frac{q}{(p - h)}. \tag{17}$$

It will be seen from (17) that the optimum rate of fertilizer is still independent of 'a', the cost of application of fertilizer per acre. However, on comparing the present result with (6), it will be noticed that in addition to q and p, the optimum rate now also depends on h, the cost of harvesting and threshing the additional produce.

Next, the M.R.R. of fertilizer is obtained, as before, by maximising the net return per rupee invested in fertilization as given by the ratio $p(Y_1 - Y_2)/pY_2$. This is equivalent to maximising

$$\frac{pY_1}{pY_2} = \frac{pf(x)}{a + qx + hf(x)}. \tag{18}$$

We shall now prove in the following theorem that the maximum of (18) is independent of 'h'.

Theorem 4.—The M.R.R. of fertilizer is independent of 'h', *i.e.*, the cost of harvesting and threshing the additional produce.

Equating the differential coefficient of the expression in (18) to zero, we find that the M.R.R. is obtained as the solution of:

$$\{a + qx + hf(x)\}\, pf'(x) - pf(x).\ \{q + hf'(x)\} = 0.$$

Cancelling out p and simplifying, this gives

$$(a + qx)\, f'(x) = qf(x)$$

or,

$$\frac{f(x)}{f'(x)} = x + \frac{a}{q} \tag{19}$$

which is independent of h, and is the same as formula (7) of Section 4. We have thus shown that the maxima of (5) and (18) are identical, and are given by the solutions of (19).

In other words, the M.R.R. of fertilizer is unaltered whether the extra cost of harvesting and threshing the additional produce is considered or not. It has thus been shown to be independent of both p and h.

6. Extension to the Case of Two or More Fertilizers

The results given in the previous sections can be easily extended to the case of two or more fertilizers. We shall state below, without proof, the main results for the case of two fertilizers.

Let the response function to x units per acre of one fertilizer (A) and y units per acre of another fertilizer (B) be represented, in units of crop yield, by the surface:

$$Z = F(x, y). \tag{20}$$

In order that the response surface (20) be concave to the xy-plane, it is necessary and sufficient that,

$$\left.\begin{array}{l}\text{(i) } rt - s^2 = \text{Positive}\\ \text{(ii) Both } r, t = \text{Negative}\end{array}\right\} \tag{21}$$

where

$$r = \frac{\partial^2 F}{\partial x^2}, \quad s = \frac{\partial^2 F}{\partial x \partial y}, \quad t = \frac{\partial^2 F}{\partial y^2}. \tag{22}$$

Let, further,

p = Price per unit of crop yield,

q_1, q_2 = Costs per unit of the two fertilizers A and B,

a = Cost of application per acre of fertilizer-mixture,

h = Cost of harvesting and threshing per md. of additional produce.

We may then easily deduce the following results for the optimum and minimum fertilization rates:—

(1) The optimum rate of fertilization is independent of 'a' and depends only on the ratios $q_1/(p-h)$ and $q_2/(p-h)$.

(2) The M.R.R. is independent of both p and h, and depends only on the ratios a/q_1 and a/q_2.

(3) For a fixed value of y, the M.R.R. x for A increases (or decreases) with a/q_1. Similarly for a fixed value of x, the M.R.R. y for B increases (or decreases) with a/q_2.

(4) For a given total investment, the total profit is maximised by using the 'minimum recommended rates' of fertilization for both the fertilizers.

7. Particular Cases and Their Solutions

Single factor response functions, concave to the fertilizer axis, and generally in use are of the following four types:—

(i) Quadratic :	$Y = bx - cx^2;\ b > o,\ c > o;$
(ii) Quadratic Square Root:	$Y = B\sqrt{x} + Cx;\ B > o;$
(iii) Cobb-Douglas:	$Y = kx^m;\ k > o,\ o < m < 1;$
(iv) Mitscherlich:	$Y = A(1 - R^x);\ A > o;\ o < R < 1;$

where, in each case, Y denotes the response over control, *i.e.*, over no fertilizer application ($x = o$). The restrictions on the constants follow simply from the fact that the response functions are positive and concave to the axis of X over the entire range under consideration.

The optimum and minimum rates of fertilization for these four forms of the response function may now be easily derived from the general solutions (17) and (19) presented in Section 5 above. The results are as follows:—

(1) *Quadratic Equation*

Optimum Rate:

$$\hat{x} = \frac{b - \dfrac{q}{p - h}}{2c}$$

Minimum Rate:

$$\breve{x} = +\sqrt{\alpha^2 - \alpha\left(\frac{b}{c}\right)} - \alpha,$$

where

$$\alpha = \frac{a}{q}.$$

(2) *Quadratic Square Root Equation*

Optimum Rate:

$$\sqrt{\hat{x}} = \frac{B}{2\left\{\dfrac{q}{p - h} - C\right\}}.$$

Minimum Rate:

$$\sqrt{\breve{x}} = +\sqrt{\alpha + \frac{\alpha^2 C^2}{B^2}} + \frac{\alpha C}{B}$$

where

$$\alpha = \frac{a}{q}.$$

(3) *Cobb-Douglas Equation*

Optimum Rate:

$$(\hat{x})^{1-m} = \frac{km}{\left(\frac{q}{p-h}\right)}.$$

Minimum Rate:

$$\breve{x} = \left(\frac{a}{q}\right)\left(\frac{m}{1-m}\right).$$

(4) *Mitscherlich Equation*

Optimum Rate:

$$R^{\hat{x}} = \frac{q}{(p-h) \,.\, A\beta}$$

Minimum Rate:

$$R^{\breve{x}} \,.\, [1 + (\breve{x} + \alpha)\,\beta] = 1$$

where

$$\alpha = \frac{a}{q}$$

and

$$\beta = -\log_e{}^{R}.$$

Since $R < 1$, $\beta > 0$.

The numerical evaluation of these rates presents no difficulty except for the minimum recommended rate in the case of the Mitscherlich response function. The corresponding equation may, however, easily be solved by iteration on starting with any approximate solution. A useful initial value to start the iteration is obviously given by the value of the M.R.R. for any other form of the response function, say, the quadratic.

8. Numerical Illustration

As a numerical illustration of the results discussed in the paper, consider the data of a fertilizer trial with wheat, conducted at the I.A.R.I. during 1949–50 and 1950–51, with a view to study the relative value of different forms of nitrogenous fertilizers (Chandnani, 1954). Four forms, namely, Urea, Chilean (or Sodium) Nitrate, Ammonium Nitrate and Ammonium Sulphate, each at 20 and 40 lb. N per acre, were tried along with Control or no fertilizer. On an average of the two years' results, the response to Urea was found to be linear and so omitted from further consideration. The average responses to the other three forms of N-fertilizers were found to be quadratic, the corresponding fitted equations being as follows:—

Ammonium Sulphate: $Y = 9\cdot44\,x - 2\cdot55\,x^2$

Ammonium Nitrate: $Y = 9\cdot27\,x - 2\cdot98\,x^2$

Chilean Nitrate: $Y = 8\cdot78\,x - 2\cdot79\,x^2$

where Y is the yield-response over control in mds. per acre and x stands for the dose of fertilizer applied per acre in units of 20 lb. N.

Both the linear and quadratic coefficients in each equation were found to be highly significant. In addition, the following two response functions were also fitted for the Ammonium Sulphate data:

Quadratic Square Root Equation (Ammonium Sulphate):

$$Y = 8\cdot706\,\sqrt{x} - 1\cdot816\,x$$

Mitscherlich Equation (Ammonium Sulphate):

$$Y = (9\cdot31)\{1 - (0\cdot26)^{x}\}.$$

The following range of prices and costs was then taken for the determination of optimum and minimum fertilization rates:—

p = Price of wheat grain per md. = Rs. 16 to Rs. 20;

q = Cost per unit of 20 lb. N of fertilizer = Rs. 12 to Rs. 18;

a = Cost of application of fertilizer per acre = Rs. 1 to Rs. 2;

h = Cost of harvesting and threshing of extra produce per md. = Rs. 2 to Rs. 4.

The range for q has been fixed on the basis of current rates for the three N-fertilizers which give a value of Rs. 17·2, 12·6 and 15·4 for a unit of 20 lb. N from Ammonium Sulphate, Ammonium Nitrate and Chilean Nitrate respectively. The ranges for the other three constants are based upon normal fluctuations in actual rates.

The variations in p, q, a and h, lead to 16 cost-price situations. The optimum and minimum rates of fertilization in all these cases have been determined with regard to the Quadratic response function for Ammonium Sulphate and the results are presented in Table I (a).

It will be seen that the results in Table I (a) are in accordance with the general properties proved earlier. The optimum rate is seen

TABLE I (a)

Optimum and Minimum Rates of Fertilization: Quadratic Response Function for Ammonium Sulphate

Situation	Prices (Rs.)				Optimum Fertilization				Minimum Fertilization			
	p	h	a	q	Rate (lb. N/acre)	Response (Md./acre)	Net profit (Rs./acre)	Profit %	Rate (lb. N/acre)	Response (Md./acre)	Net profit (Rs./acre)	Profit %
1	16	2	1	12	33·7	8·66	100·1	260	9·6	3·93	48·3	331
2	16	2	2	12	33·7	8·66	99·1	251	12·7	4·97	60·0	306
3	16	4	1	12	33·1	8·64	82·8	149	9·6	3·93	40·4	180
4	16	4	2	12	33·1	8·64	81·8	145	12·7	4·97	50·1	169
5	16	2	1	18	32·0	8·57	90·3	192	8·0	3·38	39·1	261
6	16	2	2	18	32·0	8·57	89·3	186	10·8	4·35	49·2	241
7	16	4	1	18	31·1	8·52	73·2	116	8·0	3·38	32·3	149
8	16	4	2	18	31·1	8·52	72·2	113	10·8	4·35	40·5	139
9	20	2	1	12	34·4	8·69	134·8	345	9·6	3·93	64·0	438
10	20	2	2	12	34·4	8·69	133·8	334	12·7	4·97	79·9	408
11	20	4	1	12	34·1	8·68	117·5	209	9·6	3·93	56·2	250
12	20	4	2	12	34·1	8·68	116·5	204	12·7	4·97	70·0	237
13	20	2	1	18	33·1	8·64	124·7	259	8·0	3·38	52·6	351
14	20	2	2	18	33·1	8·64	123·7	252	10·8	4·35	66·6	326
15	20	4	1	18	32·6	8·61	107·5	166	8·0	3·38	45·8	211
16	20	4	2	18	32·6	8·61	106·5	162	10·8	4·35	57·9	199

to be independent of '*a*', while the minimum rate is independent of '*p*' and '*h*'. Again, whereas the net profit per acre is higher in each case with the optimum rate, the percentage profit per rupee invested is always higher with the minimum rate of fertilization.

In order to see the effect of using a different response function on the values of the optimum and minimum rates of fertilization, these were recalculated for the last four situations of Table I (*a*), with regard to the Quadratic Square Root and Mitscherlich response functions for Ammonium Sulphate. The results are presented in Tables I (*b*) and I (*c*).

TABLE I (*b*)

Optimum and Minimum Rates of Fertilization: Quadratic Square Root Function for Ammonium Sulphate

Situation	Prices (Rs.)				Optimum Fertilization			Minimum Fertilization		
	p	*h*	*a*	*q*	Rate (lb. N/ acre)	Response (Md./ acre)	Profit %	Rate (lb. N/ acre)	Response (Md./ acre)	Profit %
13	20	2	1	18	47·8	9·12	193	1·0	1·86	561
14	20	2	2	18	47·8	9·12	188	1·9	2·53	475
15	20	4	1	18	43·8	8·91	134	1·0	1·86	298
16	20	4	2	18	43·8	8·91	131	1·9	2·53	265

TABLE I (*c*)

Optimum and Minimum Rates of Fertilization: Mitscherlich Response Function for Ammonium Sulphate

Situation	Prices (Rs.)				Optimum Fertilization			Minimum Fertilization		
	p	*h*	*a*	*q*	Rate (lb. N/ acre)	Response (Md./ acre)	Profit %	Rate (lb. N/ acre)	Response (Md./ acre)	Profit %
13	20	2	1	18	37·5	8·57	230	5·4	2·84	392
14	20	2	2	18	37·5	8·57	224	7·4	3·67	358
15	20	4	1	18	35·8	8·47	153	5·4	2·84	230
16	20	4	2	18	35·8	8·47	149	7·4	3·67	214

It would appear from these tables that while the Quadratic and Mitscherlich response functions give values for both optimum and minimum fertilization rates of a comparable nature, the Quadratic Square Root function gives widely differing results and does not appear to be suitable for the purpose.

An analysis on similar lines of the relative efficiency of the three forms of N-fertilizers, using current rates and prices and a given response function, may also be carried out. It goes to show that the net profit per acre is the highest with the optimum rate of fertilization for Ammonium Sulphate, while Ammonium Nitrate, being cheaper, gives definitely higher percentage profits.

9. SUMMARY

The concept of the 'Minimum Recommended Rate' of fertilization in relation to the 'optimum' rate and the economics of fertilizer use has only recently been introduced by Pesek and Heady (1958). The present paper derives general algebraic solutions for determining this minimum recommended rate for *any* form of the response function, in one or more variables, along with the modifications necessary when application and extra-harvesting costs are also taken into consideration. In addition, four important properties of the 'Minimum Recommended Rate' have been proved for the most general case. Explicit solutions have been given, in particular, for the various single variable response functions in common use, and the results illustrated with the help of a numerical example.

ACKNOWLEDGMENT

The author is grateful to Dr. V. G. Panse, Statistical Adviser, I.C.A.R., for going through the manuscript and making useful suggestions.

REFERENCES

1. Chandnani, J. J. .. "Preliminary studies on the relative value of different forms of commercial nitrogenous fertilizers," *Ind. Jour. Agr. Sci.*, 1954, **24**, 213–26.

2. Crowther, E. M. and Yates, F. "Fertilizer policy in war time: The fertilizer requirements of arable crops," *Emp. Jour. Expt. Agr.*, 1941, **9**, 77–97.

3. Heady, E. O., Pesek, J. T. and Brown, W. G. "Crop response surfaces and economic optima in fertilizer use," *Iowa Agr. Expt. Sta., Res. Bull.*, 1955, **424**.

4. IBACH, D. B. .. "A graphic method of interpreting response to fertilizer," *U.S.D.A., Agriculture Handbook*, 1956, **93**.

5. Mitscherlich, E. A. .. "Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages," *Landw. Jahrb.*, 1909, **38**, 537–52.

6. Panse, V. G. .. *Manuring of Cotton in India* (Revised Edition, 1953), Indian Central Cotton Committee, Bombay, 1945.

7. Paschal, J. L. .. "Economic analysis of alfalfa yield response to phosphate fertilizer," *U.S.D.A.*, 1953, FM **104**.

8. Pesek, J. T. and Heady, E. O. "Derivation and application of a method for determining minimum recommended rates of fertilization," *Soil Sci. Soc. of Amer. Proc.*, 1958, **22**, 419–23.

9. Spillman, W. J. .. "Use of the exponential yield curve in fertilizer experiments," *U.S.D.A., Tech. Bull.*, 1933, **348**,

10. Sukhatme, P. V. .. "Economics of manuring," *Ind. Jour. Agr. Sci.*, 1941, **11**, 325–37.