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Optimum Estimation of a Finite Population Total in PPS Sampling with Replacement for Multi-Character Surveys

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SUMMARY

Optimal estimator for estimating a finite population total for the probability proportional to size with replacement (PPSWR) sampling scheme under superpopulation models have been proposed. It has been shown empirically that the conventional Hansen-Hurwitz (1943) estimator fares better than the existing alternative estimators when the study variable is approximately proportional to the auxiliary variable irrespective of low correlation. However, if the study variable is approximately constant, the expansion estimator, proposed by Rao (1966) is recommended. The use of expansion estimator is also shown to be useful for linear intercept model.

Key words : PPSWR sampling, Multi-character surveys, Optimal estimator, Superpopulation model.

1. Introduction

In sampling from a finite population, the use of auxiliary information related to a study variable plays eminent role for selection of sample with varying probabilities to get an efficient estimator. In large-scale multi-character surveys, we very often estimate population parameters like totals, means and variances for more than one character at a time. Suppose a single auxiliary variable x is available and it is well related to some of the characters and poorly related to the remaining ones, in such a situation, a sample, selected with varying probabilities using the variable x as a measure of size, may provide efficient estimators for estimating population totals for those characters which are well related to the study variable but may not provide efficient estimators for the characters poorly related to the study variables. Rao (1966) first considered the problem of estimation of a finite population total under probability proportional to size with replacement (PPSWR) sampling scheme when the measure of size x is poorly related to the variable under study y. For clarity, let us formulate the problem as follows.

Let U = {1, .., i, .., N} be a finite population of N units and $y_i(x_i)$ be the value of the study (auxiliary) variable for the ith unit of the population and Y (X) be their total. The values of the auxiliary variable x_i's are assumed to be known and positive for every $i \in U$. Suppose a sample s of size n is selected by PPSWR method of sampling using normed size measure $p_i = x_i/X$, for the ith unit. The conventional Hansen-Hurwitz (1943) estimator for the population total Y is given by

$$t_0 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{0i}}$$

where $p_{0i} = p_i$.

The estimator t_0 is design unbiased (p-unbiased) for the total Y. Rao (1966) pointed out that the estimator becomes inefficient when the study variable y is poorly related to the auxiliary variable and hence he suggested an alternative biased estimator for Y as

$$t_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{1i}}$$

where $p_{1i} = 1/N$

He found that the bias of the expansion estimator t_1 is negligible and t_1 is more efficient than the conventional estimator t_0 in the sense of having smaller expected variance with respect to the following superpopulation model.

Model m1: $y_i = \mu + \varepsilon_i;\ E_m(\varepsilon_i/x_i) = 0,\ V_m(\varepsilon_i/x_i) = \sigma^2$

and $C_m(\varepsilon_i, \varepsilon_j/x_i, x_j) = 0, i \neq j$

where E_m, V_m and C_m denotes respectively expectation, variance and covariance with respect to the Model m1.

Bansal and Singh (1985) argued that correlation cannot be exactly zero in practice and hence he suggested the following alternative biased estimator t_2 involving the correlation coefficient ρ

$$t_2 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{2i}}$$

where

$$p_{2i} = \left(1+\frac{1}{N}\right)^{(1-\rho)}(1-p_i)^{\rho} - 1$$

Amahia *et al.* (1989) termed the estimator t_2 as an ad hoc one since it does not possess any desirable property except that it reduces t_0 when $\rho = 1$ and t_1 when $\rho = 0$; and proposed two alternative biased estimators t_3 and t_4 described as follows

$$t_3 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{3i}}$$

and

$$t_4 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{4i}}$$

with

$$p_{3i} = \frac{\rho p_i + (1-\rho)}{N} \text{ and } p_{4i} = \left[\frac{\rho}{p_i} + (1-\rho)N\right]^{-1}$$

It should be noted that p_{3i} and p_{4i} are respectively the weighted arithmetic mean and harmonic mean of p_i and 1/N. Amahia *et al.* showed that for the Model m1, the absolute bias of t_4 is less than that of the Rao's estimator t_1, but the bias of t_3 is less than the bias of t_1 whenever ρ is positive. As for efficiency is concerned, the estimators, t_3 and t_4 are found to be more efficient (in the sense of having less expected variance) than the conventional estimator t_0 under the superpopulation Model m1. They also tried to extend their results to a more general superpopulation model Model m2, given below, considered by Rao (1966a), Hanurav (1966) and T.J. Rao (1966) among others and ended up with some sufficient conditions which are difficult to check in practice.

Model m2: $y_i = \beta x_i + \varepsilon_i; E_m(\varepsilon_i/x_i) = 0, V_m(\varepsilon_i/x_i) = \sigma^2 x_i^g$

and $C_m(\varepsilon_i, \varepsilon_j/x_i, x_j) = 0$ for $i \neq j$ with $g \geq 0$

Mangat and Singh (1992-93) proposed the following alternative estimator analogous to Amhia *et al.* using $p_{5i} = p_i^{\rho}(1/N)^{(1-\rho)}$ as an weighted geometric mean of p_i and 1/N as

$$t_5 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{5i}} \cdot$$

Finally, Singh and Horn (1998) considered the following estimator t_6 taking into account of possible small negative correlation between the study and the auxiliary variables when the condition $\text{Max}(x_i) < X/n$ is satisfied for every

$i \in U$.

$$t_6 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_{6i}}$$

where

$$p_{6i} = (p_i^-)^{-\rho(1-\rho)/2} p_i^{\rho(1+\rho)/2} (1/N)^{(1-\rho)(1+\rho)}$$

and $$p_i^- = \frac{(X - nx_i)}{\{(N-n)X\}}$$

Singh and Horn (1998) showed empirically that their estimator becomes more efficient than the conventional estimator t_0 and the alternative estimator t_1 proposed by Rao (1966) for low values of ρ. Following Amhia *et al.*, Kumar and Agarwal (1997) proposed alternative estimators for varying probability sampling scheme.

In fact except t_1, none of the proposed alternative estimators discussed above has any desirable property. Rao's estimator t_1 is model unbiased under Model m1 in the sense $E_m(t_1) = E_m(Y) = N\mu$. We will call an estimator t as model-design $(p - \zeta)$ unbiased for Y if it satisfies $E_p E_m(t) = E_p E_m(Y)$.

The alternative estimators t_1 to t_6 were developed to use in the situation where there is a poor relationship between the study and the auxiliary variables y and x. The relationship was measured by the product moment of the correlation coefficient ρ, between x and y. It should be noted that the variables y and x may be well related (approximately proportional under Model m2) but may have low correlation. In fact in Section 3.2, it is shown that the correlation ρ is a decreasing function of g when Model m2 holds.

In this paper, optimal estimators t^* and t_g in the class of model design unbiased estimators have been proposed under Model ml and Model m2 respectively. It is worth noting that the derived optimal estimator does not involve ρ. Both the proposed optimum estimators are found superior to the estimators t_0 and t_1. The efficiencies of the optimal estimators are compared numerically with the alternative estimators through simulation studies. The simulation studies reveal that, for Model ml, estimator t_1 is as efficient as the proposed optimal estimator t^* and fares better than the rest of the alternative estimators t_2 to t_6. For Model m2, the conventional estimator t_0 is found as efficient as the optimal estimator t_g and fares better than the rest of the alternative estimators.

Finally we compare the alternative estimators under Model m3, (given in Section 3.3) which is a combination of Model m1 and Model m2 and it is shown empirically that the estimator t_g fares better than the rest and next choice is t_1.

Since the optimal estimators t^* and t_g involve unknown model parameters, the safest rule is to use the conventional estimator t_0 when Model m1 holds and to use the expansion estimator t_1 when Model m2 or Model m3 holds.

2. *Optimum Estimation of Population Total*

Let C be the class of $p-\zeta$ unbiased estimators of Y consisting of the estimators of the form

$$t=\frac{1}{n}\sum_{i=1}^{n} w_i y_i$$

where w_i's are suitably chosen constants free from y_i's and satisfying the $p-\zeta$ unbiased conditions under the Model m2 as follows

$$E_m E_p(t)=\beta\sum_{i=1}^{N} w_i x_i\ p_i=\beta X \tag{2.1}$$

An optimum estimator for Y within the class of estimators C is derived in the following theorem.

Theorem1: Under Model m2, for every $t\in C$

$$E_m V_p(t)\geq\frac{\beta^2X^2}{n}\left(\frac{1}{A}-1\right)$$

where $A=\sum_{i=1}^{N} x_i q_i p_i,\ q_i=\frac{x_i}{\{\delta^* x_i^g(1-p_i)+x_i^2\}},\ \delta^*=\left(\frac{\sigma^2}{\beta^2}\right)$

Equality is attained if t equals

$$t_g=\frac{1}{n}\sum_{i=1}^{n} w_{i0}y_i,\ w_{i0}=q_i X/A$$

Proof (sketch).

$$E_m V_p(t)=E_m\frac{1}{n}\left[\sum_{i=1}^{N} w_i^2 y_i^2 p_i-\left(\sum_{i=1}^{N} w_i y_i p_i\right)^2\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{N} w_i^2\left\{\sigma^2 x_i^g p_i(1-p_i) + \beta^2 x_i^2 p_i\right\} - \beta^2\left(\sum_{i=1}^{N} x_i w_i p_i\right)^2\right] \tag{2.2}$$

Now minimizing (2.2) with respect to w_i subject to $p-\zeta$ unbiasedness condition (2.1), the optimum value of w_i is obtained as $w_{i0} = q_i X/A$.

Finally putting the optimum value of $w_i = w_{i0}$ in (2.2), the expression of $E_m V_p(t_g)$ is obtained.

Remark 1 : The optimal estimator t_g involves unknown model parameters β, σ and g. So, the estimator t_g is usable in practice when good knowledge of the model parameters are obtained either from the past experience or can be estimated through the data collected.

Model m2 reduced to the Model m1 when $x_i = 1$ and $\beta = \mu$. Hence putting $x_i = 1$ and $\beta = \mu$, in Theorem 1, the optimum estimator under Model m1 is obtained and is given in the following theorem.

Theorem 2 : Under Model m1, for every t satisfying $E_m E_p(t) = \mu$

$$E_m V_p(t) \ge \frac{N^2\mu^2}{n}\left(\frac{1}{A_0} - 1\right) = E_m V_p(t^*)$$

where

$$t^* = \frac{1}{n}\sum_{i=1}^{n} w_{i0} y_i,\ w_{i0} = q_{i0}\, N / A_0$$

$$A_0 = \sum_{i=1}^{N} q_{i0} p_i$$

$$q_{i0} = \frac{1}{\{\delta(1-p_i)+1\}},\ \delta = \frac{\sigma^2}{\mu^2}$$

Remark 2 : The optimal estimator t^* is usable when good knowledge of δ, square of the coefficient of variation for the study variable y under Model m1 is available either from the past experience or from the data collected.

Remark 3 : The optimal estimator t^* reduces to t_1 when $\delta = 0$. Hence t_1 becomes optimal when μ is sufficiently large compared to σ.

Since estimator t_0 is p (design) unbiased and t_1 is ζ (model) unbiased under Model m1 and Model m2 both the estimators t_0 and t_1 belong to the class

C of $p-\zeta$ unbiased estimators. The estimators t^* and t_l are optimal in the class C under the Model m1 and Model m2 respectively and hence more efficient than both the estimators t_0 and t_1. These can be summarized in the following theorems.

Theorem 4 : Under Model m1, $E_m V_p(t^*) \le E_m V_p(t_j)$ for $j = 0$ and 1

Theorem 5 : Under Model m2, $E_m V_p(t_g) \le E_m V_p(t_j)$ for $j = 0$ and 1

The expressions for bias and variance of t_k for k = 0, ..., 6 are obtained as follows

$$\text{Variance of } t_k = V_p(t_k) = \frac{1}{n}\left[\sum_{i=1}^{N} \frac{y_i^2 p_i}{p_{ki}^2} - \left(\sum_{i=1}^{N} \frac{y_i p_i}{p_{ki}}\right)^2\right]$$

$$\text{Bias of } t_k = B_p(t_k) = \sum_{i=1}^{N} y_i\left(\frac{p_i}{p_{ki}} - 1\right)$$

3. *Comparison of Efficiency*

Since the estimators t_j's for j = 2, ..., 6 do not belong to the class C and expressions for $E_m V_p(t_j)$ are complex, no meaningful theoretical comparison is possible. So, relative efficiency $E_k = \left[\frac{V_p(t_0)}{V_p(t_k)}\right] \times 100$ and absolute bias $|B_p(t_k)|$, of t_k for $k = 0, \ldots, 6$ along with the optimal estimators are compared under Model m1, Model m2 and Model m3 through simulation studies and presented in Table 1, Table 2 and Table 3.

3.1 Comparison under Model m1

In order to study relative efficiencies of the estimators described above, we generate five populations each of size 500 (= N). For each of the population, the auxiliary variable x was kept fixed and it was obtained by taking random sample from a gamma population with parameters $\alpha = 10$ and $\beta = 1$. To generate the study variable y, we first select ε_i a random sample from a normal population with mean zero and standard deviation σ, $[N(0, \sigma)]$ and then y_i is obtained using the relation $y_i = \mu + \varepsilon_i$, for $i = 1, \ldots 500$. Five populations are considered by taking different values of μ and σ. The relative efficiencies are presented in the following Table 1.

Table 1. Efficiency under Model m1

Population		t_1	t_2	t_3	t_4	t_5	t_6	t^*	t_r
1	E_k	4248.36	3631.00	3630.15	3667.58	3735.46	4094.99	4234.97	3745.25
	$\rho = -.0749$								
	$\mu = 100$								
	$\varepsilon \sim N(0, 5)$								
	$\lvert B_k \rvert$	60.33	330.54	331.01	64.85	128.71	81.69	81.69	.63
2	E_k	1354.91	1340.04	1340.02	1351.62	1346.90	1350.23	1354.08	1242.22
	$\rho = -.0194$								
	$\mu = 100$								
	$\varepsilon \sim N(0, 10)$								
	$\lvert B_k \rvert$	32.58	80.99	81.14	33.21	22.00	10.66	10.66	.89
3	E_k	1049.08	1049.52	1049.52	1048.82	1049.21	1049.34	1047.58	951.41
	$\rho = .0074$								
	$\mu = 50$								
	$\varepsilon \sim N(0, 5)$								
	$\lvert B_k \rvert$	5.94	11.61	11.63	5.90	2.65	.95	.95	.18
4	E_k	11812.72	11167.13	11166.68	10991.48	11198.55	11603.91	11818.74	10688.71
	$\rho = .0277$								
	$\mu = 100$								
	$\varepsilon \sim N(0, 3)$								
	$\lvert B_k \rvert$	12.70	111.29	111.43	12.35	49.63	37.51	37.51	.16
5	E_k	24195.25	21739.7	217380.22	21661.5	22004.58	23336.61	24188.72	22046.01
	$\rho = -.0235$								
	$\mu = 100$								
	$\varepsilon \sim N(0, 2)$								
	$\lvert B_k \rvert$	7.53	104.11	104.24	7.71	48.04	35.52	35.52	.14

From Table 1, we note that all the estimators are better than the conventional estimator t_0. For the Populations 1, 2, 4 and 5, t_1 and t^* are found more efficient than the rest of the estimators t_2 to t_6, and for the Population 3 all are equally efficient. It is worth noting that there is no significant difference in efficiencies for the estimators t_1 and t^*. As for the bias is concerned, the estimators t_2 and t_3 have higher absolute bias than the rest.

The above estimators are also compared with the following alternative ζ – unbiased ratio estimator

$$t_r = N\frac{\sum_{i=1}^{n}\frac{y_i}{p_i}}{\sum_{i=1}^{n}\frac{1}{p_i}}$$

It is found that the ratio estimator t_r has minimum absolute bias in each of the four populations. As per efficiency concerned, the ratio estimator is less efficient than all other estimators discussed here.

3.2 Comparison under Model m2

The expected value of product moment of correlation coefficient $\rho = \frac{S_{xy}}{(S_{xx}.S_{yy})^{1/2}}$ under Model m2 is given by

$$E_m(\rho) = \beta\frac{S_{xx}}{\left[\sigma^2\sum_{1}^{N}x_i^g\left(1-\frac{1}{N}\right)+\beta S_{xx}\right]^{1/2}}$$

where $S_{uv} = \frac{\sum_{1}^{N}u_iv_i - \sum_{1}^{N}u_i\sum_{1}^{N}v_i}{N}$; u, v = x, y

From the expression of $E_m(\rho)$, we note that the correlation is a decreasing function of g under the model m2 if x_i's > 1 for every i.

To generate the study and auxiliary variable we proceed as follows. First we take a random sample x_i from N(15, 1), then we select sample ε_i from $N(0, \sigma^* x_i^{g/2})$ with $\sigma^* = 5$ and finally y_i is obtained using the relation $y_i = \beta x_i + \varepsilon_i$. The procedure is repeated for 500 (= N) times to generate a population with a particular value of β and g. In all we generate 11 populations each of size 500 (=N) with the same $\beta = 5$ but 11 different values of g given, in Table 2 along with the correlation coefficients.

From Table 2, we note that efficiency of all the estimators except the optimal estimator t_g is less than 100 for all values of ρ. It is also worth noting that efficiency of the optimal estimator t_g is very close to 100 in all the 11 populations. Thus we conclude that the conventional estimator is always more efficient than the other alternative estimators described in this paper. The performance of the optimal estimator is virtually same as the optimum estimator t_g. It should be noted that t_1 is least efficient than the rest for $\rho(g) \leq .2176(1.25)$. For $\rho(g) \geq .2176(1.25)$, and all the alternative estimators t_1 to t_6 are equally efficient.

As for bias is concerned, t_g has a practically no bias in all the 11 populations as long as $g \leq 2$ and t_1 has larger absolute bias relative to each of the rival alternatives. For g = 4 each of the estimator got large absolute bias.

Table 2. Efficiency under Model m2

Population		t_1	t_2	t_3	t_4	t_5	t_6	t_g
1	E_k	38.49	92.12	92.12	91.95	92.05	87.82	99.96
	$\rho = .7832$							
	$g = 0$							
	$\|B_k\|$	173.02	8.82	8.79	37.50	23.14	29.19	.01
2	E_k	49.76	89.31	89.31	88.97	89.15	84.99	99.82
	$\rho = .7123$							
	$g = .25$							
	$\|B_k\|$	190.29	20.03	19.99	54.74	37.35	46.97	.19
3	E_k	69.58	91.96	91.96	91.82	91.90	88.62	99.99
	$\rho = .5475$							
	$g = .50$							
	$\|B_k\|$	168.05	34.26	34.22	76.04	55.11	67.41	.01
4	E_k	79.89	92.48	92.48	92.16	92.32	90.01	99.97
	$\rho = .4343$							
	$g = .75$							
	$\|B_k\|$	173.70	56.04	56.00	98.25	77.11	91.56	.03
5	E_k	89.73	95.19	95.19	94.99	95.09	93.89	100.01
	$\rho = .3214$							
	$g = 1.0$							
	$\|B_k\|$	167.56	82.22	82.18	119.08	100.99	115.57	.16
6	E_k	95.38	97.36	97.36	97.22	97.29	96.79	100.06
	$\rho = .2176$							
	$g = 1.25$							
	$\|B_k\|$	165.53	101.03	101.00	129.50	115.26	126.95	.11

7	E_k	97.35	98.28	98.28	98.17	98.22	97.98	100.07
	$\rho = .1582$							
	$g = 1.50$							
	$\|B_k\|$	162.93	114.45	114.42	137.15	125.81	135.98	.53
8	E_k	97.57	98.13	98.13	98.02	98.07	97.92	100.03
	$\rho = .1114$							
	$g = 1.75$							
	$\|B_k\|$	156.85	122.89	122.87	139.37	131.13	138.17	.61
9	E_k	98.79	98.95	98.96	98.91	98.93	98.89	100
	$\rho = .058$							
	$g = 2$							
	$\|B_k\|$	123.87	107.73	107.73	116.68	112.20	115.56	.03
10	E_k	97.03	96.88	98.88	96.91	96.90	96.95	101.06
	$\rho = -.0341$							
	$g = 3$							
	$\|B_k\|$	267.27	271.56	271.59	276.14	273.97	271.02	372.27
11	E_k	97.48	97.34	97.34	97.37	97.36	97.40	100.84
	$\rho = -.0367$							
	$g = 4$							
	$\|B_k\|$	1185.98	121.84	1215.82	1229.52	122.47	1207.31	3296.47

3.3 Comparison under Model m3

Model m3 : $y_i = \mu + \beta x_i + \varepsilon_i$ may be regarded as a combination of the Model m1 and Model m2. For Model m3, we first take a random sample x_i from N(25, 5) for each x_i and a given value of ρ, we select y_i from $N(\mu_{xi}, 5)$, with $\mu_{xi} = 100 + \rho(x_i - 25)$ for $i = 1, \ldots, 500$. Thus the generated data (y_i, x_i) becomes a random sample of size 500 from a bivariate normal population with correlation coefficient ρ. We choose 10 different values of ρ's to generate 10 different populations. The efficiencies of the estimators are given in Table 3. From Table 3, we note that in all the 10 populations, the proposed optimum estimator t^* has the maximum efficiency and all the alternative estimators fare better than the conventional estimator t_0. The estimator t_1 is found to be more efficient than the rest of the alternative estimators t_2 to t_6, t_6 is found to be more efficient than t_2, t_3, t_4, and t_5; t_2 and t_3 are found to be equally efficient in all the populations and better than t_4 and t_5; t_5 is better than t_4.

As for bias is concerned, all the alternative estimators have very small biases compared to standard error. t_1 and t^* have the same amount of bias in all the populations. t_6 has the lowest absolute bias for $\rho \leq .4549$ whereas t_5 has the least bias for $\rho \geq .5220$. t_2 and t_3 have approximately same amount of bias in all the situations.

Table 3. Efficiency under Model m3

Population		t_1	t_2	t_3	t_4	t_5	t_6	t^*
1	E_k	1367.44	1265.99	1265.99	1224.67	1253.28	1352.95	1370.90
	$\rho = .1345$							
	$\lvert B_k \rvert$	66.15	146.13	146.33	57.25	46.08	28.94	65.97
2	E_k	1707.26	1227.05	1227.06	1067.81	1175.27	1513.24	1716.00
	$\rho = .2433$							
	$\lvert B_k \rvert$	131.32	294.92	295.30	99.36	102.83	79.49	130.97
3	E_k	1368.96	875.64	875.67	716.49	812.91	1131.54	1379.33
	$\rho = .3394$							
	$\lvert B_k \rvert$	186.09	333.96	334.39	122.91	110.92	93.90	185.52
4	E_k	1286.80	669.91	669.91	610.72	647.87	930.41	1296.46
	$\rho = .3888$							
	$\lvert B_k \rvert$	200.44	332.54	332.98	122.50	107.84	98.27	199.92
5	E_k	993.67	548.58	548.61	471.96	513.78	738.43	1003.64
	$\rho = .4549$							
	$\lvert B_k \rvert$	268.35	375.49	347.97	146.27	103.40	98.86	267.45
6	E_k	1074.95	456.25	456.27	391.52	426.20	625.69	1088.58
	$\rho = .5220$							
	$\lvert B_k \rvert$	318.12	368.47	386.99	152.05	119.65	125.53	317.03
7	E_k	1039.30	375.73	375.77	310.87	342.96	495.91	1054.69
	$\rho = .5966$							
	$\lvert B_k \rvert$	370.80	384.72	385.22	149.56	118.23	134.24	369.50
8	E_k	814.22	338.14	338.17	293.04	314.93	445.18	827.61
	$\rho = .6161$							
	$\lvert B_k \rvert$	418.91	364.46	364.96	160.81	102.22	118.78	417.19
9	E_k	592.59	269.18	269.20	239.33	253.11	341.73	604.29
	$\rho = .6875$							
	$\lvert B_k \rvert$	494.09	302.78	303.22	154.36	73.31	93.14	491.69
10	E_k	593.39	265.99	266.03	217.34	238.99	319.90	604.92
	$\rho = .7191$							
	$\lvert B_k \rvert$	518.19	286.30	286.70	145.51	66.67	86.68	515.79

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