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Testimation of Error Variance for a Three–way Layout in Random Effects Model under Asymmetric Loss Function

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SUMMARY

The present paper proposes a testimation procedure using a preliminary test of significance for the estimation of the error variance for three–way layout to be used for testing a main effect in the random effects model. The risk properties of this testimation procedure have been studied under asymmetric loss function and it is observed through numerical computations that the estimator for error variance performs better than the estimator under 'Squared Error Loss Function (SELF)' for certain range of the nuisance parameters and certain combinations of degrees of freedom. Recommendations for the choice of degree of asymmetry and the level of significance have been made, the procedure presents a method as how, to control the error with proper choices of these quantities.

Key words : Three–way layout, Random effects model, Preliminary test, Squared error loss function, Asymmetric loss function, Risk, Nuisance parameters.

1. INTRODUCTION

Suppose that an agricultural equipment(s) producing concern is producing some small parts to be used in the equipments say sprayer etc. The parts are being produced using a large number of machines of same make and model. The concern may be interested in getting an answer to the question: 'Is there any difference between the machines?" Since the total number of machines in use is very large, it is not possible to make such a study by taking samples of output of all machines. Therefore, keeping this and other related problems in mind the following experiment is performed.

A random sample of I machines from the lot of machines and J workers from the totality of workers has been selected independently. Each worker is assigned to work on a machine for one day. A random sample of K batches of materials produced by each worker on a machine from the total output is selected. Since for any machine or worker or batch of material there may be considerable variation we will treat the total output as though it is a continuous random variable.

The effect of using preliminary test of significance (PTS) on subsequent estimation was first considered by Bancroft (1944) who investigated the bias and mean square error (MSE) of a variance estimator obtained after performing a PTS of equality of two variances. The effects of using a PTS on size and power of ANOVA test procedures have been studied by Paull (1956), Bechofer (1951a, b), Bozivich *et al.* (1956), Srivastava and Bozivich (1961) and Gupta (1965). Gupta and Singh (1976) studied the bias and MSE of the proposed estimator which can be used for testing the main effects for three–way layout in random effects model. The importance of the said study was the fact that unless the estimator is good it may not give good results in final testing. Such studies were earlier made by Srivastava and Gupta (1965) and Gupta and Srivastava (1969) for fixed effects model.

The object of the present investigation is to study the risk properties of the sometimes pool estimation procedure for error variance in random effects model using asymmetric loss function. The motivation behind this study is that asymmetric loss function has been used by many workers in the context of life testing models for estimating or testimating the parameters such as mean life, guarantee period, reliability functions etc. and it has been observed that the estimators obtained using asymmetric loss function perform better than those obtained under squared error loss function. Srivastava and Rao (1992) have dealt with the estimation of disturbance variance in linear regression models under asymmetric loss function. They have studied the performance properties of the conventional estimators for the disturbance (variance) in the model and have compared that with the performance of estimator using under asymmetric loss function. All the studies using asymmetric loss function have demonstrated their superiority over the conventional studies (i.e. studying the estimators under squared error loss function).

The present study is an attempt to apply the idea of using asymmetric loss function in the context of ANOVA models.

In Bayesian estimation, statistical inference is made when we are given a model, a distribution of parameters and a loss function associated with the decision we make for the parameter. Under this set up an experimenter expresses his belief about the real situation via a prior distribution and misjudgment by the loss function.

For various life testing models, it has been observed that in the estimation of mean life or reliability function the use of squared error loss function (SELF) may be inappropriate as has been pointed out by Canfield (1970), Varian (1975), Zellner (1986), Basu and Ebrahimi (1991), Pandey and Rai (1992, 1996) and Srivastava (1996). The above mentioned authors and many others have suggested to use asymmetric loss function for estimating or testimating the parameters or parametric functions in the context of life testing models. However, Srivastava and Tanna (2001) have studied the risk properties of error variance estimator in random effects model. Varian (1975) proposed asymmetric loss function, which has been found to be appropriate in the situations where either over-estimation is more serious than the under-estimation or vice–versa. Over-estimation / under-estimation of error variance may lead to unwarranted values of F – calculated which in turn affects the acceptance/ rejection of hypothesis of interest. If we can control this situation, it will be of much practical usage. Use of LINEX loss function facilitates this, the way it is defined. The proper choice of 'a' makes it possible.

The asymmetric loss function is given by

$$L(\Delta)=b\left(e^{a\Delta}-a\Delta-1\right),\ a\neq 0,\ b>0 \qquad (1.1)$$

where, $\Delta=\left(\hat{\theta}-\theta\right)$ or $\Delta=\left(\frac{\hat{\theta}}{\theta}-1\right)$

depending upon parameter (i.e. location or scale) which is being estimated.

The sign and magnitude of 'a' represents the direction and degree of asymmetry respectively. The positive value of 'a' is used when over-estimation is more serious than under-estimation, while a negative value of 'a' is used in reverse situations. $L(\Delta)$ defined in (1.1) involves the terms like $a\Delta$, $a\Delta^2$, $a\Delta^3$,.... the value (magnitude) of 'a' will also affect these terms and that will be depending upon positive/ negative values of 'a', which depends upon over-estimationt/ under-estimation in that sense 'a' represents asymmetry and its value(s) the degree. $L(\Delta)$ rises exponentially when $\Delta<0$ and almost linearly when $\Delta>0$. 'b' is the factor of proportionality and it is 'a' which determines the relative losses for positive and negative values of estimation error. However, for 'a' close to zero, this loss function is approximately squared error loss function and almost symmetric. Throughout the present study we have taken b=1.

2. STATEMENT OF THE PROBLEM

Let Y_{ijkl} denotes the l^{th} observation in the k^{th} batch of material produced by j^{th} worker if he uses i^{th} machine. The sample observations can well be represented by a complete three–way layout, designating machines as factor A, workers as factor B and batches as factor C. Thus, we can assume that

$$Y_{ijkl}=\mu+a_i^A+a_j^B+a_k^C+a_{ij}^{AB}+a_{jk}^{BC}+a_{ik}^{AC}+a_{ijk}^{ABC}+e_{ijkl}$$

$$i=1,\ldots,I;\ j=1,\ldots,J;\ k=1,\ldots K;\ l=1,\ldots,L$$

The random variables a_i^A are uncorrelated and have $N\left(0,\sigma_A^2\right)$ distributions. Similarly a_j^B have

$N(0, \sigma_B^2), \ldots a_{ijk}^{ABC}$ have $N(0, \sigma_{ABC}^2)$ distributions. The error e_{ijkl} are independently and identically distributed with mean zero and variance σ_e^2. We are interested in testing the main hypothesis $H_A : \sigma_A^2 = 0$ against the alternative $H'_A : \sigma_A^2 > 0$ i.e. we are interested in examining whether there is any significant difference between the machines from which these I machines have been drawn at random beyond their variation from l[th] one batch to another or in their use by different workers. It may be pointed out here that the test for $\sigma_B^2 = 0$ or $\sigma_C^2 = 0$ can be accomplished by the above test procedure by simply relabelling the main effects (i.e. renaming B as A, etc.) and hence a detailed study of testing the hypothesis that $\sigma_A^2 = 0$ only has been made. We note that to test H_A no exact test is available unless we assume that either of the two, two factor interactions be zero. Since and Gupta (1976) have proposed an estimate of error variance assuming that the interaction AB may or may not be zero and AC is greater than zero, then (1.1) can be written as

$$\begin{aligned} Y_{ijkl} &= \mu + a_i^A + a_j^B + a_k^C + a_{ij}^{AB} + a_{jk}^{BC} + a_{ik}^{AC} \\ &\quad + a_{ijk}^{ABC} + e_{ijkl} \quad \text{if } \sigma_{AC}^2 > 0 \text{ and } \sigma_{AB}^2 > 0 \\ Y_{ijkl} &= \mu + a_i^A + a_j^B + a_k^C + a_{jk}^{BC} + a_{ik}^{AC} \\ &\quad + a_{ijk}^{ABC} + e_{ijkl} \quad \text{if } \sigma_{AC}^2 > 0 \text{ and } \sigma_{AB}^2 = 0 \end{aligned} \tag{2.2}$$

We are given three mean squares V_1, V_2, V_3 representing the source of variations i.e. interaction(s) due to the factors ABC, AC and AB respectively based upon n_1, n_2, n_3 degrees of freedom respectively. Let $E(V_i) = \sigma_i^2$, $(i = 1,2,3)$, and $\sigma_2^2 > \sigma_1^2$ and $\sigma_3^2 \geq \sigma_1^2$. Now for a particular analysis we may require an estimator of $\sigma^2 = \sigma_3^2 + \sigma_2^2 - \sigma_1^2$. One obvious estimator which can be suggested is $V_3 + V_2 - V_1 = V_A$ (say). This estimator is unbiased also. If however, $\sigma_3^2 = \sigma_1^2$ then one can also see that V_2 is an unbiased estimator of σ^2. The problem is that which one is to be selected. This type of problem arises in practice when testing the main effects for three or higher–way layout in random effects model. If we use the former estimator i.e. V_A, we get an approximate F–test since no exact test is available. The estimator of V_A has been given by Satterthwaite (1941) and its use in testing the main effects has been suggested by Scheffe (1959, pp. 247 – 248). However, the use of V_2 provides an exact test.

The mean squares V_i's, $(i = 1,2,3 \text{ and } 4)$ are independently distributed as $\frac{\sigma_i^2 \chi_i^2}{n_i}$ where χ_i^2 is a central chi-square statistic with n_i degrees of freedom.

It follows from Table 1.1 that the estimator of $\sigma^2 = \sigma_{ABC}^2 + K\sigma_{AB}^2 + J\sigma_{AC}^2$ will be $V_A = V_3 + V_2 - V_1$ which can be used for testing H_A. If we can assume that $\sigma_{AB}^2 = 0$ then the estimate of error variance is V_2.

Finally, if we assume that $\sigma_{AB}^2 \geq 0$ then we have an uncertainty about the model specification. The uncertainty can be resolved by first testing the hypothesis $H: \sigma_{AB}^2 = 0$ against the alternative $H': \sigma_{AB}^2 > 0$. The model finally chosen and hence the estimator finally selected for the analysis depends upon the outcome of this original test known as the preliminary test.

Table 1.1. Analysis of variance for three–way layout in random effects model

Sources of Variation	Degrees of Freedom	Mean Squares	Expected Mean Squares
A	$n_4 = (I-1)$	V_4	$\sigma_4^2 = \sigma_e^2 + L\sigma_{ABC}^2 + KL\sigma_{AB}^2 + JL\sigma_{AC}^2 + JKL\sigma_A^2$
AB	$n_3 = (I-1)(J-1)$	V_3	$\sigma_3^2 = \sigma_e^2 + L\sigma_{ABC}^2 + KL\sigma_{AB}^2$
AC	$n_2 = (I-1)(K-1)$	V_2	$\sigma_2^2 = \sigma_e^2 + L\sigma_{ABC}^2 + JL\sigma_{AC}^2$
ABC	$n_1 = (I-1)(J-1)(K-1)$	V_1	$\sigma_1^2 = \sigma_e^2 + L\sigma_{ABC}^2$

Therefore, we use the following rule procedure for the estimation of the error variance

$$V_{SP}=\begin{cases} V_2 & \text{if } \frac{V_3}{V_1}\le F(n_3,n_1,\alpha) \\ V_A & \text{if } \frac{V_3}{V_1}> F(n_3,n_1,\alpha) \end{cases}$$

where, $V_A=V_3+V_2-V_1$ and $F(n_3,n_1,\alpha)$ is upper $100\alpha\%$ point of the F- distribution with (n_3,n_1) degrees of freedom.

It may be noted that V_A will always be positive since it is used as an estimator of error variance only, if $\frac{V_3}{V_1}>F(n_3,n_1,\alpha)$ since $F(n_3,n_1,\alpha)\ge 1$ always, we have $\frac{V_3}{V_1}\ge 1$ and $\frac{V_2}{V_1}>0$. Hence $\frac{V_3}{V_1}+\frac{V_2}{V_1}>1$, thus, $V_3+V_2-V_1>0$ i.e. V_A is always positive.

We derive the risk of estimator V_{SP} if we follow the above rule of procedure.

3. DERIVATION OF RISK OF THE ESTIMATOR V_{SP}

The sum of squares, $\frac{n_iV_i}{\sigma_i^2},(i=1,2,3)$ are independently distributed as central $\chi^2_{n_i}$ with n_i degrees of freedom. The joint probability density function of V_1,V_2,V_3 is given by

$$f(V_1,V_2,V_3)=CV_1^{\frac{n_1}{2}-1}V_2^{\frac{n_2}{2}-1}V_3^{\frac{n_3}{2}-1}\exp\left[-\frac{1}{2}\left(\frac{n_1V_1}{\sigma_1^2}+\frac{n_2V_2}{\sigma_2^2}+\frac{n_3V_3}{\sigma_3^2}\right)\right]dV_1dV_2dV_3 \quad (3.1)$$

$$\text{where, } C=\frac{\left(\frac{n_1}{\sigma_1^2}\right)^{\frac{n_1}{2}}\left(\frac{n_2}{\sigma_2^2}\right)^{\frac{n_2}{2}}\left(\frac{n_3}{\sigma_3^2}\right)^{\frac{n_3}{2}}}{2^{\frac{n_{123}}{2}}\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)\Gamma\left(\frac{n_3}{2}\right)}$$

and $n_{123}=n_1+n_2+n_3$

The risk of V_{SP} under $L(\Delta)$ is given by

$$R_{SP}=E\left[L(V_{SP},\sigma^2)\right]=E\left[L(V_2,\sigma^2)\Big/\frac{V_3}{V_1}\le\beta\right]Pr\left(\frac{V_3}{V_1}\le\beta\right)+E\left[L(V_A,\sigma^2)\Big/\frac{V_3}{V_1}>\beta\right]Pr\left(\frac{V_3}{V_1}>\beta\right) \quad (3.2)$$

$$R_{SP}=E_1 Pr\left[\frac{V_3}{V_1}\le\beta\right]+E_2 Pr\left(\frac{V_3}{V_1}>\beta\right)$$

where, $E_1=E\left[L(V_2,\sigma^2)\Big/\frac{V_3}{V_1}\le\beta\right]$

$E_2=E\left[L(V_A,\sigma^2)\Big/\frac{V_3}{V_1}>\beta\right]$ and

$\beta=f(n_3,n_1,\alpha)$

To calculate the expectations, let us make the following transformations in (3.1)

$$u_1=\frac{n_1V_1}{\sigma^2},u_2=\frac{n_2V_2}{n_1V_1}\theta_{12},u_3=\frac{n_3V_3}{n_1V_1}\theta_{13}$$

where, $\theta_{13}=\frac{\sigma_1^2}{\sigma_3^2},\theta_{12}=\frac{\sigma_1^2}{\sigma_2^2}$ and $\beta'=\frac{n_3}{n_1}\theta_{13}\beta$

On substitution we get

$$f(u_1,u_2,u_3)=Cu_1^{\frac{n_{123}}{2}-1}u_2^{\frac{n_2}{2}-1}u_3^{\frac{n_3}{2}-1}\exp\left[-\frac{1}{2}u_1(1+u_2+u_3)\right]du_1du_2du_3 \quad (3.3)$$

$$\text{where, } C=\frac{1}{2^{\frac{n_{123}}{2}}\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)\Gamma\left(\frac{n_3}{2}\right)}$$

Now

$$R_{SP} = \int_{u_1=0}^{\infty}\int_{u_2=0}^{\infty}\int_{u_3=0}^{\beta'}\left[e^{a\left(\frac{V_2}{\sigma^2}-1\right)} - a\left(\frac{V_2}{\sigma^2}-1\right)-1\right] g(u_1,u_2,u_3)du_1du_2du_3$$

$$+ \int_{u_1=0}^{\infty}\int_{u_2=0}^{\infty}\int_{u_3=\beta'}^{\infty}\left[e^{a\left(\frac{V_A}{\sigma^2}-1\right)} - a\left(\frac{V_A}{\sigma^2}-1\right)-1\right] g(u_1,u_2,u_3)du_1du_2du_3 \quad (3.4)$$

The straight forward integration of (3.4) leads us to

$$R_{SP} = \left[\frac{a}{\theta^*\theta_{13}} I_{X_0}\left(\frac{n_3}{2}+1, \frac{n_1}{2}\right) - \frac{a}{\theta^*} I_{X_0}\left(\frac{n_3}{2}, \frac{n_1}{2}+1\right) + \frac{e^{-a} I_{X_0}\left(\frac{n_3}{2}, \frac{n_1}{2}\right)}{\left(1-\frac{2a}{n_2\theta^*\theta_{12}}\right)^{\frac{n_2}{2}}} + \frac{e^{-a}\left[1-I_{Y_0}\left(\frac{n_3}{2}, \frac{n_1}{2}\right)\right]}{\left(1+\frac{2a}{n_1\theta^*}\right)^{\frac{n_1}{2}}\left(1-\frac{2a}{n_2\theta^*\theta_{12}}\right)^{\frac{n_2}{2}}\left(1-\frac{2a}{n_3\theta^*\theta_{13}}\right)^{\frac{n_3}{2}}} - 1\right] \quad (3.5)$$

where

$$X_0 = \frac{n_3\theta_{13}\beta'}{n_1 + n_3\theta_{13}\beta'}, \quad Y_0 = \frac{\left(1-\frac{2a}{n_3\theta^*\theta_{13}}\right)\beta'}{\left(1+\frac{2a}{n_1\theta^*}\right)+\left(1-\frac{2a}{n_3\theta^*\theta_{13}}\right)\beta'}$$

$$\theta^* = (\theta_{31} + \theta_{21} - 1)$$

4. RISK COMPARISON

A natural way of comparing the behaviour of the proposed estimator is to examine the performance of it with respect to the best available estimator for this purpose. We define the relative risk of V_{SP} with respect to V_A as follows

$$R_R = \frac{R_N}{R_{SP}} \quad (4.1)$$

where R_N is the risk of the estimator V_A of σ^2 under $L(\Delta)$ and V_A is always unbiased estimator of σ^2 so, it seems more appropriate to talk of the relative risk of V_A to V_{SP} under $L(\Delta)$.

Now

$$R_N = \int_{u_1=0}^{\infty}\int_{u_2=0}^{\infty}\int_{u_3=0}^{\infty}\left[e^{a\left(\frac{V_A}{\sigma^2}-1\right)} - a\left(\frac{V_A}{\sigma^2}-1\right)-1\right] g(u_1,u_2,u_3)du_1\,du_2\,du_3 \quad (4.2)$$

A straightforward integration of (4.2) gives

$$R_N = \frac{e^{-a}}{\left(1+\frac{2a}{n_1\theta^*}\right)^{\frac{n_1}{2}}\left(1-\frac{2a}{n_2\theta^*\theta_{12}}\right)^{\frac{n_2}{2}}\left(1-\frac{2a}{n_3\theta^*\theta_{13}}\right)^{\frac{n_3}{2}}} - 1 \quad (4.3)$$

Hence

R_R =[expression (4.3)][expression (3.5)]$^{-1}$

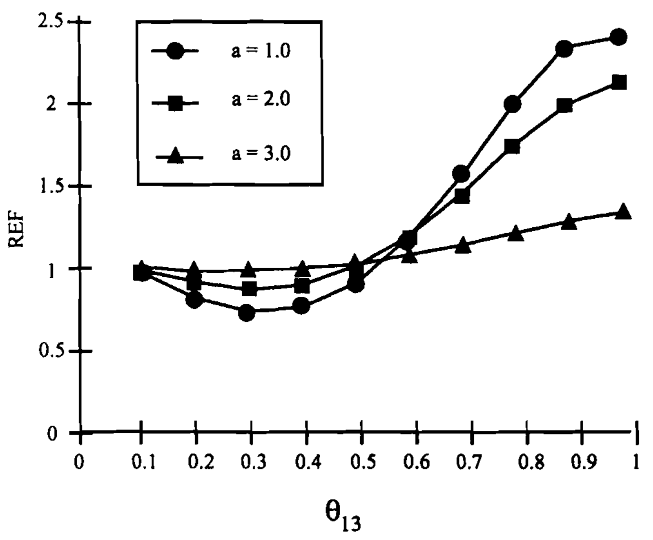
Evidently R_R is a function of $n_1, n_2, n_3, \theta_{13}, \theta_{12}, \theta^*, \alpha$ and 'a'. In which n_1, n_2, n_3 are taken by experimenter as per convenience, $\theta_{13}, \theta_{12}, \theta^*$ are the nuisance parameters, giving the values $\theta_{13}, \theta_{12}, \theta^*$ is mathematically determined. We have fixed some values of θ_{12} i.e. θ_{12} = 0.1, 0.5, 1.0 and have allowed the variation in θ_{13} from 0.1(0.1)1.0 as θ_{13} is the uncertainty in terms of $\sigma_3^2 \geq \sigma_1^2$. We have taken five data sets for n_1, n_2, n_3 i.e. (i) $n_1 = 24$, $n_2 = 12, n_3 = 8$ (ii) $n_1 = 24$, $n_2 = 8, n_3 = 12$ (iii) $n_1 = 34, n_2 = 8, n_3 = 14$ (iv) $n_1 = 45, n_2 = 7, n_3 = 16$ (v) $n_1 = 100, n_2 = 55, n_3 = 79$.

The values of α^S have been taken as 1%, 5%, 10% and 25%. We have selected positive as well as negative values of 'a' i.e. a=1, 2, 3 and −1, −2, −3 to observe the effect of degree/direction of 'a'. For all the above values relative risks have been computed and some graphs are provided. However, our conclusions based on all the graphs are given in the next section.

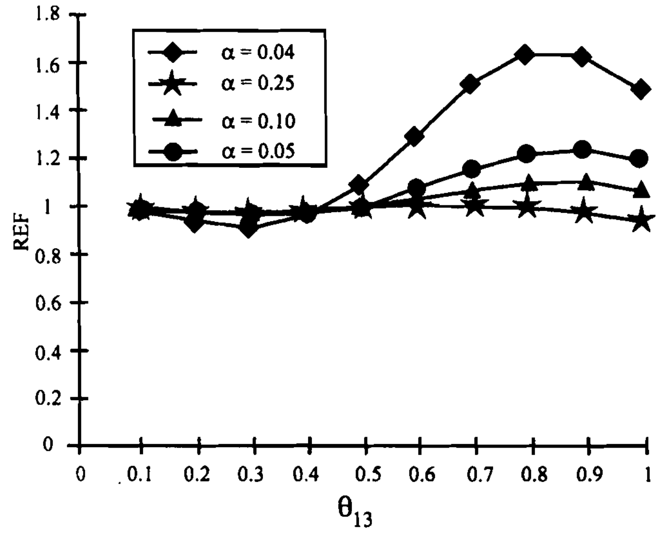
Graphs of Relative Risks for Selected Values of Level of Significance, Degree of Asymmetry

1. $n_1 = 24$, $n_2 = 12$, $n_3 = 8, \alpha = 0.01$, $\theta_{12} = 1.0$

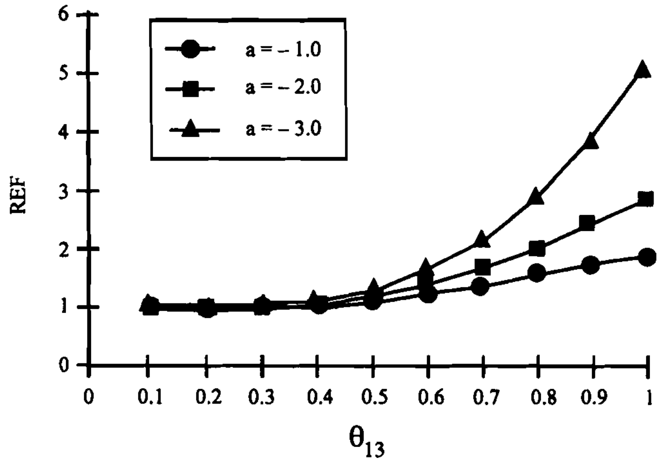
REF (relative risk)



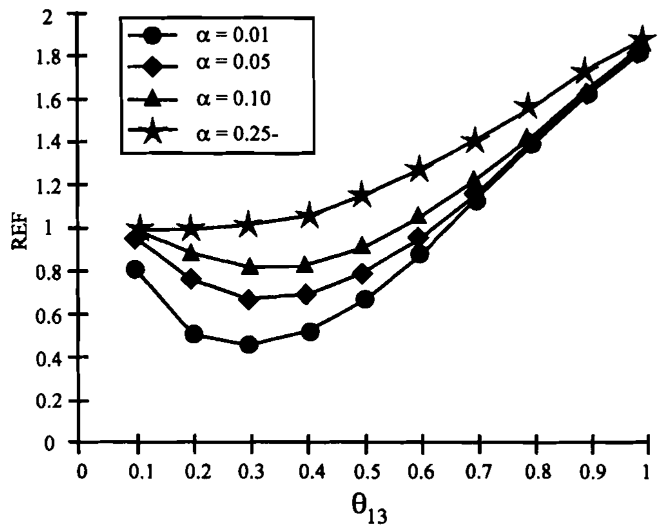
2. $n_1 = 24$, $n_2 = 12$, $n_3 = 8$, $a = 3.0$, $\theta_{12} = 0.5$



3. $n_1 = 24$, $n_2 = 12$, $n_3 = 8$, $\alpha = 0.25$, $\theta_{12} = 1.0$

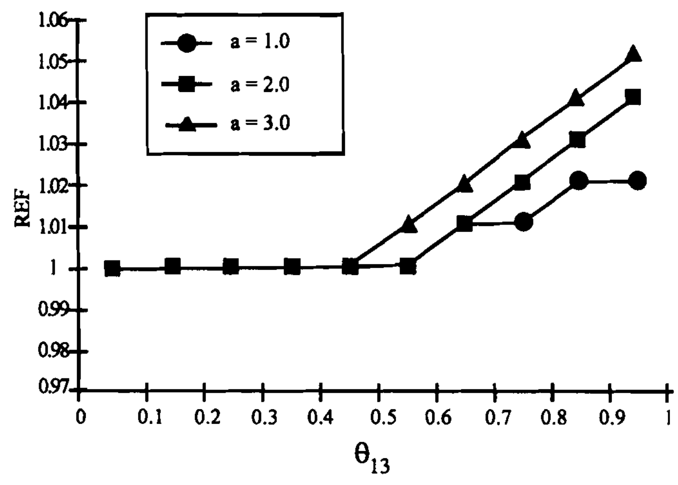


4. $n_1 = 24$, $n_2 = 12$, $n_3 = 8$, $a = -3.0$, $\theta_{12} = 0.5$



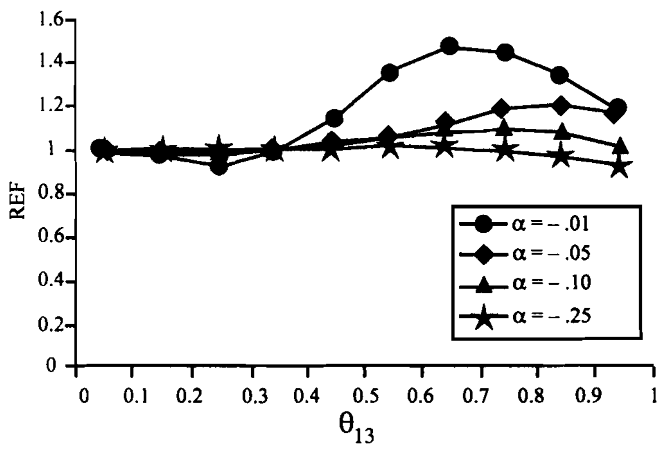
5. $n_1 = 34$, $n_2 = 8$, $n_3 = 14$, $\alpha = 0.01$, $\theta_{12} = 0.1$

REF (relative risk)



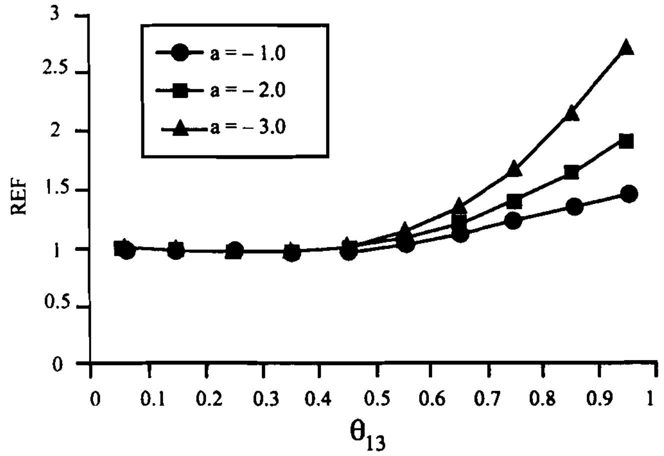
6. $n_1 = 45,\ n_2 = 7,\ n_3 = 16,\ a = 3.0,\ \theta_{12} = 0.5$

REF(relative risk)



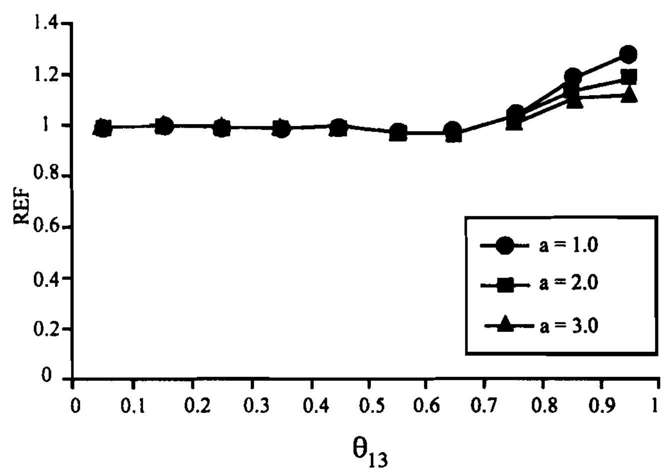
7. $n_1 = 45,\ n_2 = 7,\ n_3 = 16,\ \alpha = 0.25,\ \theta_{12} = 1.0$

REF(relative risk)



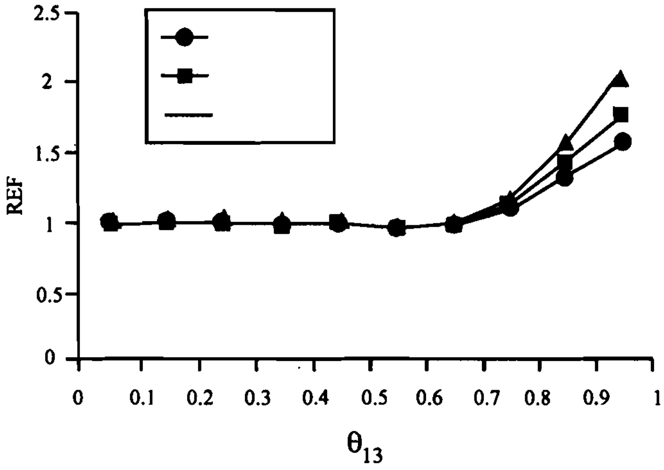
8. $n_1 = 100,\ n_2 = 55,\ n_3 = 79,\ \alpha = 0.25,\ \theta_{12} = 1.0$

REF(relative risk)



9. $n_1 = 100,\ n_2 = 55,\ n_3 = 79,\ \alpha = 0.25,\ \theta_{12} = 1.0$

REF(relative risk)



5. CONCLUSION

We have computed the values of R_R (Relative Risks) for different combinations of values of n_1, n_2 and n_3 mentioned above. As it can be seen from the values of $n_i^s, (i = 1,2,3)$ need not be a multiple of 4^S. It has been observed that proposed testimator of error variance fairs better than unbiased estimator of the same for almost the whole range of θ_{12} i.e. $\theta_{12} = 0.1, 0.5, 1.0$, however, for θ_{13} when $\theta_{12} = 0.1$, the range is $\theta_{13} > 0.5$. Further, when $\theta_{12} = 0.5$ and 1.0, it is the whole range θ_{13} i.e. from 0.1(0.1)1.0 where V_{SP} performs better for a wide range of values of n_i^S. In the present study the quantities of attention were the level(s) of significance and the degree of asymmetry as this controls the over-estimation/ under-estimation. For this purpose we have fixed α at 1% and have allowed the variation(s) in 'a' (a=1, 2, 3) and it is observed that the performance of V_{SP} is better than V_A for all the values of 'a' and it is at its best at a=1.0, however its performance is better for other values of 'a' also considered here.

Next we have fixed a=3 (reason being to consider the impact of higher order of asymmetry) and to decide about α^S, it is seen from the R_R values that α =1% the performance of V_{SP} is the best. Again we have considered the negative value of 'a' (a= –1, –2, –3) and have allowed variations in the values of α and it is observed that the highest magnitude of R_R is obtained at α =10% at a = –3.0. For other values of α^S and a^S the R_R values indicate that V_{SP} almost dominates over V_A.

In the present study it is concluded that a lower value of α i.e. $\alpha = 1\%$ yields better results when over-estimation is more serious than under-estimation (i.e. at a=1.0). However a little higher value of $\alpha = 10\%$ is recommended for reverse situation i.e. a= –3.0.

Looking at the above discussion, where the proposed estimator fairs better than V_A, it can be seen that the superiority of the proposed estimator holds wherever $R_R > 1$, within the ranges of 'a', α and $n_i, (i = 1,2,3)$ considered here, it can be concluded that the proposed estimator does not perform uniformly better than V_A.

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