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On the use of Several Auxiliary Variates to Improve the Precision of Estimates at Current Occasion

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SUMMARY

The present work is an attempt to make use of several auxiliary variates at both the occasions for improving the precision of estimates at current occasion in two occasions successive sampling. Chain-type difference and regression estimators have been proposed for estimating the population mean at current occasion in two occasions rotation (successive) sampling. The proposed estimators have been compared with sample mean estimator when there is no matching and the optimum successive sampling estimator when no additional auxiliary information has been used. Optimum replacement policy is discussed. Theoretical results have been justified through empirical means of elaboration.

Key words: Successive sampling, Auxiliary variates, Chain-type, Variance, Bias, Mean square error, Optimum replacement policy.

1. INTRODUCTION

There are many problems of practical interest in different fields of applied and environmental sciences in which the various characters opt to change over time with respect to different parameters. Hence, one is often concerned with measuring the characteristics of a population on several occasions to estimate the trend in time of population means as a time series or the current value of population mean or the value of population mean over several points of time. For example, an investigator or owner of the industry of cold drinks may be interested in the following types of problems: (a) The average or total sale of cold drink for the current season; (b) The change in average sale of cold drink for two different seasons; or (c) Simultaneously to know both (a) and (b).

The follow-up of objective is carried out by means of sampling on successive occasions (over years or seasons or months) according to a specified rule, with partial replacement of units, called successive (rotation) sampling. Successive (rotation) sampling provides a strong tool for generating the reliable estimates at different occasions. Theory of rotation (successive) sampling appears to have started with the work of Jessen (1942). He pioneered using the entire information collected in the previous investigations (occasions). This theory was extended by Patterson (1950), Rao and Graham (1964), Gupta (1979), Das (1982), among others. Sen (1971) developed estimators for the population mean on the current occasion using information on two auxiliary variates available on previous occasion. Sen (1972, 1973) extended his work for several auxiliary variates. Singh *et al*. (1991), and Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two occasions successive sampling. Singh (2003) extended their work for h-occasions successive sampling. In many situations, information on an auxiliary variate may be readily available on the first as well as on the second occasion, for example tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation, number of polluting industries and vehicles are known in environmental survey. Many other situations in biological (life) sciences could be explored to show the benefits of the present study. Utilizing the auxiliary information on both the occasions Feng and Zou (1997), Biradar and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007) have proposed varieties of chain-type ratio and difference estimators for estimating the population mean at current (second) occasion in two occasions successive sampling.

Following the work of Singh and Priyanka (2007) the objective of the present work is to propose estimators for estimating population mean at current occasion using several auxiliary variates. In order to provide an in-depth presentation of the proposed work, an illustrative scenario is provided. Consider the case of public health and welfare of a state or country. In that case several instances are available that can be treated as auxiliary variates, such as the number of beds in different hospitals may be known, number of doctors and supporting staffs may be available, the amount of funds available for medicine etc. may be known. Likewise there are several informations available, which if efficiently utilized can improve the precision of estimates. Two estimators have been suggested and its theoretical properties are discussed. Empirical results indicate the dominance of proposed estimators over other existing estimators.

2. FORMULATION OF ESTIMATORS

2.1 Notations

Let $U = \left(U_1, U_2, \ldots, U_N\right)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by x (y) on the first (second) occasions respectively. It is assumed that information on p (non negative integer constant) auxiliary variates z_j, j = 1, 2, ..., p whose population means are known, closely related (positively correlated) to x and y on the first and second occasions respectively, available on the first as well as on the second occasion. For convenience, it is assumed that the population under consideration is considerably large enough. A simple random sample (without replacement) of n units is taken on the first occasion. A random sub-sample of $m = n\lambda$ units is retained (matched) for use on the second occasion, while a fresh simple random sample (without replacement) of $u = (n - m) = n\mu$ units are drawn on the second occasion from the remaining (N – n) units of the population so that the sample size on the second occasion is also n. λ and μ ($\lambda + \mu = 1$) are the fractions of matched and fresh samples respectively at the second (current) occasion. The following notations have been considered for the further use.

$\bar{X}, \bar{Y}$: Population means of the study variate x and y respectively.

$\bar{Z}_j$: Population mean of the j^{th} (j = 1, 2, ..., p) auxiliary variate.

$\bar{x}_n, \bar{x}_m, \bar{y}_u, \bar{y}_m$: Sample means of the respective variates of the sample sizes shown in suffices.

$\bar{z}_{uj}, \bar{z}_{nj}, \bar{z}_{mj}$: Sample means of j^{th} (j = 1, 2, ..., p) auxiliary variate of sample sizes shown in suffices.

$\rho_{yx}, \rho_{xzj}, \rho_{yzj}, \rho_{zjzk}$: Correlation coefficients between the variates shown in suffices, where $j \neq k = 1, 2, ..., p$.

S_x^2, S_y^2, S_{zj}^2 : Population mean squares of x, y and z_j respectively, where j = 1, 2, ..., p.

2.2 Estimator based on Population Regression Coefficients

To estimate the population mean on the second occasion, utilizing information on p auxiliary variates, two different estimators are suggested. One is a difference estimator based on sample of size $u = (n\,\mu)$ drawn afresh on the second occasion and is given by

$$T_1 = \bar{y}_u + \sum_{j=1}^{p} \beta_{yzj}\left(\bar{Z}_j - \bar{z}_{uj}\right) \tag{1}$$

Second estimator is a chain-type difference to difference estimator based on the sample of size $m\ (= n\,\lambda)$ common with both the occasions and is defined as

$$T_2 = \bar{y}_m^* + \beta_{yx}\left(\bar{x}_n^* - \bar{x}_m^*\right) \tag{2}$$

where
$$\bar{y}_m^* = \bar{y}_m + \sum_{j=1}^{p} \beta_{yzj}\left(\bar{Z}_j - \bar{z}_{mj}\right)$$

$$\bar{x}_n^* = \bar{x}_n + \sum_{j=1}^{p} \beta_{xzj}\left(\bar{Z}_j - \bar{z}_{nj}\right)$$

$$\bar{x}_m^* = \bar{x}_m + \sum_{j=1}^{p} \beta_{xzj}\left(\bar{Z}_j - \bar{z}_{mj}\right)$$

and β_{yx}, β_{xzj} and β_{yzj} (j = 1, 2, ..., p) are known population regression coefficients between the variates shown in suffices.

Combining the estimators T_1 and T_2, we have the final estimator of $\bar{Y}$ as

$$T = \varphi T_1 + (1 - \varphi) T_2 \tag{3}$$

where φ is an unknown constant to be determined under certain criterion.

2.3 Estimator based on Sample Regression Coefficients

Replacing the unknown regression parameters by their consistent estimates, we get the following working version of the estimator defined in equation (3).

$$T^* = \psi T_1^* + (1 - \psi) T_2^* \tag{4}$$

where $$T_1^* = \bar{y}_u + \sum_{j=1}^{p} b_{yzj}(u)\left(\bar{Z}_j - \bar{z}_{uj}\right)$$

and $$T_2^* = \bar{y}_m^{**} + b_{yx}(m)\left(\bar{x}_n^{**} - \bar{x}_m^{**}\right)$$

$$\bar{y}_m^{**} = \bar{y}_m + \sum_{j=1}^{p} b_{yzj}(m)\left(\bar{Z}_j - \bar{z}_{mj}\right)$$

$$\bar{x}_n^{**} = \bar{x}_n + \sum_{j=1}^{p} b_{xzj}(n)\left(\bar{Z}_j - \bar{z}_{nj}\right)$$

$$\bar{x}_m^{**} = \bar{x}_m + \sum_{j=1}^{p} b_{xzj}(m)\left(\bar{Z}_j - \bar{z}_{mj}\right)$$

ψ is an unknown constant to be determined so that it minimizes the mean square error of the estimator T^*. $b_{yzj}(u)$, $b_{yx}(m)$, $b_{yzj}(m)$, $b_{xzj}(n)$ and $b_{xzj}(m)$ are the sample regression coefficients between the variates shown in suffices and based on the sample sizes indicated in brackets.

Remark 2.1: If p = 1, estimators T and T^* defined in equations (3) and (4) respectively reduces to Singh and Priyanka (2007) estimators.

3. BIAS AND VARIANCE (MEAN SQUARE ERROR) OF PROPOSED ESTIMATORS

3.1 Bias and Variance of the Estimator T

Theorem 3.1.1. T is unbiased estimator of $\bar{\square}$.

Proof. Since, T_1 and T_2 are respectively the difference type estimators and they are unbiased for $\bar{\square}$. The final estimator T is a convex linear combination of T_1 and T_2, therefore T is also an unbiased estimator of $\bar{Y}$.

Theorem 3.1.2. The variance of the estimator T is given by

$$V(T) = \varphi^2 V(T_1) + (1 - \varphi)^2 V(T_2) \tag{5}$$

$$V(T_1) = \frac{1}{u} A\, S_y^2 \tag{6}$$

$$V(T_2) = \left[\frac{1}{m} A + \left(\frac{1}{m} - \frac{1}{n}\right) B\right] S_y^2 \tag{7}$$

where $$A = 1 - \sum_{j=1}^{p} \rho_{yzj}^2 + \sum_{j \neq k = 1}^{p} \rho_{yzj}\, \rho_{yzk}\, \rho_{zjzk}$$

and $$B = 2\rho_{yx} \sum_{j=1}^{p} \rho_{yzj}^2 - \rho_{yx}^2 \left(1 + \sum_{j=1}^{p} \rho_{yzj}^2\right) + \left(\rho_{yx}^2 - 2\rho_{yx}\right) \sum_{j \neq k = 1}^{p} \rho_{yzj}\, \rho_{jzk}\, \rho_{zjzk}$$

Proof. It is clear that the variance of T is given by

$$V(T) = \varphi^2 V(T_1) + (1-\varphi)^2 V(T_2) + 2\varphi(1-\varphi)\mathrm{Cov}(T_1, T_2) \quad (8)$$

Variance and covariance terms of equation (8) can be derived as

$$V(T_1) = E\left[T_1 - \bar{Y}\right]^2$$

$$= \left(\frac{1}{u} - \frac{1}{N}\right)\left[1 - \sum_{j=1}^{p} \rho_{yzj}^2 + \sum_{j \neq k=1}^{p} \rho_{yzj}\,\rho_{yzk}\,\rho_{zjzk}\right] S_y^2$$

$$= \left(\frac{1}{u} - \frac{1}{N}\right) A\, S_y^2 \quad (9)$$

$$V(T_2) = E\left[T_2 - \bar{Y}\right]^2$$

$$= E\left[\left(\bar{y}_m^* - \bar{Y}\right) + \beta_{yx}\left\{\left(\bar{x}_n^* - \bar{X}\right) - \left(\bar{x}_m^* - \bar{X}\right)\right\}\right]^2$$

Taking expectation we get the variance of T_2 as

$$V(T_2) = \left[\left(\frac{1}{m} - \frac{1}{N}\right) A + \left(\frac{1}{m} - \frac{1}{n}\right) B^*\right] S_y^2 \quad (10)$$

where $$B^* = 2\rho_{yx} \sum_{j=1}^{p} \rho_{xzj}\,\rho_{yzj} - \rho_{yx}^2\left(1 + \sum_{j=1}^{p} \rho_{xzj}^2\right) + \left(\rho_{yx}^2 - 2\rho_{yx}\right) \sum_{j \neq k=1}^{p} \rho_{xzj}\,\rho_{xzk}\,\rho_{zjzk}$$

Since, the estimators T_1 and T_2 are unbiased estimators and based on two independent samples of sizes u and m respectively, hence

$$\mathrm{Cov}(T_1, T_2) = 0 \quad (11)$$

Further, we consider the following assumptions:

(i) Population size is sufficiently large (i.e., $N \to \infty$), therefore finite population corrections (fpc) are ignored.

(ii) "$\rho_{xzj} = \rho_{yzj}$", $\forall$ j = 1, 2, ..., p. This is an intutitive assumption, which has been considered by Cochran (1977) and Feng and Zou (1997).

In the light of above assumptions the equations (9) and (10) take the following form

$$V(T_1) = \frac{1}{u} A\, S_y^2 \quad (12)$$

$$V(T_2) = \left[\frac{1}{m} A + \left(\frac{1}{m} - \frac{1}{n}\right) B\right] S_y^2 \quad (13)$$

Now substituting the values of $V(T_1)$, $V(T_2)$ and $\mathrm{Cov}(T_1, T_2)$ from equations (12), (13) and (11) respectively in equation (8) we get V(T) as in equation (5).

3.2 Bias and Mean Square Error of T^*

Since, T_1^* and T_2^* are the simple linear regression type and chain-type regression in regression estimators respectively, they are biased for $\bar{Y}$. Therefore, the resulting estimator T^* defined in equation (4) is also biased estimator of $\bar{Y}$. The bias B (.) and mean square error M (.) up-to the first order of approximations and for large population of size $N \to \infty$ (ignoring fpc) are derived under the following transformations and using large sample approximations:

$\bar{y}_u = \bar{Y}(1 + e_1)$ $\quad$ $\bar{y}_m = \bar{Y}(1 + e_2)$

$\bar{x}_n = \bar{X}(1 + e_3)$ $\quad$ $\bar{x}_m = \bar{X}(1 + e_4)$

$s_{yx}(m) = S_{yx}(1 + e_5)$ $\quad$ $s_x^2(m) = S_x^2(1 + e_6)$

$s_{yzj}(u) = S_{yzj}(1 + e_{7j})$ $\quad$ $s_{yzj}(m) = S_{yzj}(1 + e_{7j}^*)$

$s_{zj}^2(u) = S_{zj}^2(1 + e_{8j})$ $\quad$ $s_{zj}^2(m) = S_{zj}^2(1 + e_{8j}^*)$

$s_{zj}^2(n) = S_{zj}^2(1 + e_{8j}^{**})$ $\quad$ $s_{xzj}(n) = S_{xzj}(1 + e_{9j})$

$$\bar{z}_{uj} = \bar{Z}_j\left(1+e_{10j}\right) \qquad \bar{z}_{mj} = \bar{Z}_j\left(1+e_{11j}\right)$$

$$\bar{z}_{nj} = \bar{Z}_j\left(1+e_{12j}\right)$$

such that $|e_i| < 1$; $|e_{lj}| < 1$, $|e^{*}_{l'j}| < 1$ and $|e^{**}_{8j}| < 1$, $\forall$ i = 1, 2, 3, 4, 5, 6; j = 1, 2, ..., p; l = 7, 8, ..., 12 and l′ = 7, 8.

Under the above transformations T_1^* and T_2^* take the following forms

$$T_1^* = \left[\bar{Y}\left(1+e_1\right) - \sum_{j=1}^{p}\beta_{yzj}\,\bar{Z}_j\,e_{10j}\left(1+e_{7j}\right)\left(1+e_{8j}\right)^{-1}\right] \tag{14}$$

$$\begin{aligned} T_2^* = {} & \bar{Y}\left(1+e_2\right) - \sum_{j=1}^{p}\beta_{yzj}\,\bar{Z}_j\,e_{11j}\left(1+e^*_{7j}\right)\left(1+e^*_{8j}\right)^{-1} \\ & + \beta_{yx}\left(1+e_5\right)\left(1+e_6\right)^{-1}\left[\bar{X}\left(e_3 - e_4\right)\right. \\ & + \sum_{j=1}^{p}\beta_{xzj}\bar{Z}_j\left\{e_{11j}\left(1+e^*_{9j}\right)\left(1+e^*_{8j}\right)^{-1}\right. \\ & \left.\left. - e_{12j}\left(1+e_{9j}\right)\left(1+e^{**}_{8j}\right)^{-1}\right\}\right] \end{aligned} \tag{15}$$

Thus, we have the following theorems.

Theorem 3.2.1. The bias of the estimator T^* in estimating the population mean $\bar{Y}$, to the first order of approximations is

$$B\left(T^*\right) = \psi\, B\left(T_1^*\right) + (1-\psi) B\left(T_2^*\right) \tag{16}$$

where
$$B\left(T_1^*\right) = -\left(\frac{1}{u}\right)\sum_{j=1}^{p}\beta_{yzj}\left[\frac{C_{012}}{S_{yzj}} - \frac{C_{003}}{S^2_{zj}}\right] \tag{17}$$

and
$$\begin{aligned} B\left(T_2^*\right) = {} & -\frac{1}{m}\sum_{j=1}^{p}\beta_{yzj}\left[\frac{C_{012}}{S_{yzj}} - \frac{C_{003}}{S^2_{zj}}\right] \\ & + \left(\frac{1}{m} - \frac{1}{n}\right)\left[\beta_{yx}\left\{\frac{C_{300}}{S^2_x} - \frac{C_{210}}{S_{yx}}\right\}\right. \\ & \left. + \beta_{yx}\sum_{j=1}^{p}\beta_{xzj}\left\{\frac{C_{111}}{S_{yx}} - \frac{C_{201}}{S^2_x} - \frac{C_{003}}{S^2_{zj}} + \frac{C_{102}}{S_{xzj}}\right\}\right] \end{aligned} \tag{18}$$

where $C_{rst} = E\left[\left(x_i - \bar{X}\right)^r\left(y_i - \bar{Y}\right)^s\left(z_{ij} - \bar{Z}_j\right)^t\right]$; $\square \geq \circ$, $\square \geq \circ$, $\square \geq \circ$, j = 1, 2, ..., p.

Proof. $B\left(T^*\right) = E\left[T^* - \bar{Y}\right] = \psi\, B\left(T_1^*\right) + (1-\psi) B\left(T_2^*\right)$

where $B\left(T_1^*\right) = E\left[T_1^* - \bar{Y}\right]$

$$= E\left[\bar{Y}\left(1+e_1\right) - \sum_{j=1}^{p}\beta_{yzj}\,\bar{Z}_j\,e_{10j}\left(1+e_{7j}\right)\left(1+e_{8j}\right)^{-1} - \bar{Y}\right]$$

Expanding the right hand side of the above expression binomially, taking expectations and collecting the terms upto the order o(n^{-1}), we have

$$B\left(T_1^*\right) = -\left(\frac{1}{u} - \frac{1}{N}\right)\sum_{j=1}^{p}\beta_{yzj}\left[\frac{C_{012}}{S_{yzj}} - \frac{C_{003}}{S^2_{zj}}\right] \tag{19}$$

Similarly

$$B\left(T_2^*\right) = E\left[T_2^* - \bar{Y}\right]$$

$$\begin{aligned} = E\Bigg[& \bar{Y}\left(1+e_2\right) - \sum_{j=1}^{p}\beta_{yzj}\,\bar{Z}_j\,e_{11j}\left(1+e^*_{7j}\right)\left(1+e^*_{8j}\right)^{-1} \\ & + \beta_{yx}\left(1+e_5\right)\left(1+e_6\right)^{-1}\left[\bar{X}\left(e_3 - e_4\right) + \sum_{j=1}^{p}\beta_{xzj}\bar{Z}_j\right. \\ & \left\{e_{11j}\left(1+e^*_{9j}\right)\left(1+e^*_{8j}\right)^{-1}\right. \\ & \left.\left. - e_{12j}\left(1+e_{9j}\right)\left(1+e^{**}_{8j}\right)^{-1}\right\}\right] - \bar{Y}\Bigg] \end{aligned}$$

Again expanding the right hand side of the above expression binomially, taking expectations and retaining the terms upto the first order of approximations we have

$$B\left(T_2^*\right) = -\left(\frac{1}{m}-\frac{1}{N}\right)\sum_{j=1}^{p}\beta_{yzj}\left[\frac{C_{012}}{S_{yzj}}-\frac{C_{003}}{S_{zj}^2}\right]$$

$$+\left(\frac{1}{m}-\frac{1}{n}\right)\left[\beta_{yx}\left\{\frac{C_{300}}{S_x^2}-\frac{C_{210}}{S_{yx}}\right\}\right.$$

$$\left.+\beta_{yx}\sum_{j=1}^{p}\beta_{xzj}\left\{\frac{C_{111}}{S_{yx}}-\frac{C_{201}}{S_x^2}-\frac{C_{003}}{S_{zj}^2}+\frac{C_{102}}{S_{xzj}}\right\}\right] \tag{20}$$

Applying $N \to \infty$ in equations (19) and (20) we get the expressions for the bias of the estimators T^*, T_1^*, and T_2^* upto the first order of approximations as shown in equations (16), (17) and (18) respectively.

Theorem 3.2.2. The mean square error of the estimator T^* of the population mean $\bar{Y}$, to the first order of approximations, is given by

$$M\left(T^*\right) = \psi^2 M\left(T_1^*\right)+(1-\psi)^2 M\left(T_2^*\right) \tag{21}$$

where $$M\left(T_1^*\right) = \frac{1}{u}A\, S_y^2 \tag{22}$$

and $$M\left(T_2^*\right) = \left[\frac{1}{m}A+\left(\frac{1}{m}-\frac{1}{n}\right)B\right]S_y^2 \tag{23}$$

Proof. By the definition of mean square error we have

$$M(T^*) = E\left[T^*-\bar{Y}\right]^2$$

$$= E\left[\psi\left(T_1^*-\bar{Y}\right)+(1-\psi)\left(T_2^*-\bar{Y}\right)\right]^2$$

$$= \psi^2 M\left(T_1^*\right)+(1-\psi)^2 M\left(T_2^*\right)$$

$$+\,2\psi(1-\psi)E\left[\left(T_1^*-\bar{Y}\right)\left(T_2^*-\bar{Y}\right)\right] \tag{24}$$

where $$M\left(T_1^*\right) = E\left[T_1^*-\bar{Y}\right]^2$$

and $$M\left(T_2^*\right) = E\left[T_2^*-\bar{Y}\right]^2$$

Now, using the expressions given in equations (14) and (15), expanding binomially, taking expectations, retaining the terms upto the first order of approximations we have the following results

$$M\left(T_1^*\right) = \left(\frac{1}{u}-\frac{1}{N}\right)A\, S_y^2 \tag{25}$$

$$M\left(T_2^*\right) = \left[\left(\frac{1}{m}-\frac{1}{N}\right)A+\left(\frac{1}{m}-\frac{1}{n}\right)B^*\right]S_y^2 \tag{26}$$

In the light of assumptions used in the proof of Theorem 3.1.2, equations (25) and (26) take the following forms

$$M\left(T_1^*\right) = \frac{1}{u}A\, S_y^2 \tag{27}$$

$$M\left(T_2^*\right) = \left[\frac{1}{m}A+\left(\frac{1}{m}-\frac{1}{n}\right)B^*\right]S_y^2 \tag{28}$$

Since, the estimators T_1^* and T_2^* are biased estimators and based on two independent samples of sizes u and m respectively, therefore for large population ($N \to \infty$) we have

$$E\left[\left(T_1^*-\bar{Y}\right)\left(T_2^*-\bar{Y}\right)\right] = 0 \tag{29}$$

Substituting the values of $M\left(T_1^*\right)$, $M\left(T_2^*\right)$ and $E\left[\left(T_1^*-\bar{Y}\right)\left(T_2^*-\bar{Y}\right)\right]$ from equations (27), (28) and (29) into equation (24) we get the value of $M(T^*)$ as shown in equation (21).

Remark 3.2.1. From equations (21)-(23), it is visible that the mean square error upto the first order of approximations of the estimator T^* is exactly similar to that of the variance of the estimator T in equations (5)-(7). This is one of the positive aspects of the estimator T^*. The estimator T^* is based on sample estimates and upto the first order of approximations, it is equally precise as well as T.

4. MINIMUM VARIANCE (MEAN SQUARE ERROR) OF PROPOSED ESTIMATOR

4.1 Minimum Variance of T

Since, the variance of T in equation (5) is a function of unknown constant φ, therefore, it is minimized with respect to φ and subsequently the optimum value of φ is obtained as

$$\varphi_{opt.} = \frac{V(T_2)}{V(T_1) + V(T_2)} \tag{30}$$

Substituting the value of $\varphi_{opt.}$ in equation (5), we get the optimum variance of T as

$$V(T)_{opt.} = \frac{V(T_1).V(T_2)}{V(T_1)+V(T_2)} \tag{31}$$

Further, substituting the values from equations (6) and (7) in equations (30) and (31) respectively the simplified values of $\varphi_{opt.}$ and $V(T)_{opt.}$ are shown in Theorem 4.1.1.

Theorem 4.1.1. $\varphi_{opt.} = \dfrac{\mu[A + \mu B]}{[A + \mu^2 B]}$ (32)

and $$V(Y)_{opt.} = \frac{A[A + \mu B]}{[A + \mu^2 B]} \frac{S_y^2}{n} \tag{33}$$

Corollary 4.1.1. If there is no matching, i.e., μ = 1 then

$$V(T)_{opt.} = \frac{A}{n} S_y^2 \tag{34}$$

Corollary 4.1.2. If there is complete matching, i.e., μ = 0 then

$$V(T)_{opt.} = \frac{A}{n} S_y^2 \tag{35}$$

In both the cases $V(T)_{opt.}$ has the same value, which is the variance of the difference estimator under the assumption $N \to \infty$. This gives an implication that there must be an optimum choice of μ, other than extreme values so that $V(T)_{opt.}$ will be smaller than the quantity given in equations (34) or (35). Thus, for making current estimate (neither the case of "complete matching" nor the case of "no matching") better, it is always preferable to replace the sample partially.

4.2 Minimum Mean Square Error of T*

Since, mean square error of T^* derived in equation (21) is a function of ψ , it could be minimized with respect to ψ and we get the optimum value of ψ as

$$\psi_{opt.} = \frac{M(T_2^*)}{M(T_1^*) + M(T_2^*)} \tag{36}$$

Substituting $\psi_{opt.}$ from equation (36) into equation (21) we get

$$M(T^*)_{opt.} = \frac{M(T_1^*).M(T_2^*)}{M(T_1^*) + M(T_2^*)} \tag{37}$$

Again substituting the values from equations (22) and (23) in equations (36) and (37) we get the optimum values of ψ and mean square error of T^* upto the first order of approximations. Since, the mean square error of the estimators T_1^* and T_2^* upto the first order of approximations derived in equations (22) and (23) are coinciding with the expressions of the variances of the estimators T_1 and T_2 given in equations (6) and (7) respectively, hence, upto the first order of approximations, the values of $\psi_{opt.}$ and $M(T^*)_{opt.}$ in equations (36) and (37) will be similar to the expressions of $\varphi_{opt.}$ and $V(T)_{opt.}$ derived in equations (32) and (33) respectively.

5. OPTIMUM REPLACEMENT POLICY

5.1 Optimum Replacement Policy for the Estimator T

To determine the optimum value of μ so that $\bar{Y}$ may be estimated with maximum precision, we minimize $V(T)_{opt.}$ in equation (33) with respect to μ, which results in a quadratic equation in μ, shown as

$$B\,\mu^2 + 2\,A\,\mu - A = 0 \tag{38}$$

Solving equation (38) for μ, the solutions are given as

$$\hat{\mu} = \frac{-A \pm \sqrt{A(A+B)}}{B} \tag{39}$$

The real values of $\hat{\mu}$ exists if $(A + B) \geq 0$. For any combinations of correlations, which satisfies this condition, two real values of $\hat{\mu}$ are possible, hence to choose a value of $\hat{\mu}$, it should be remembered that $0 \leq \hat{\mu} \leq 1$, all other values of $\hat{\mu}$ are inadmissible. Substituting the admissible value of $\hat{\mu}$ say μ_0 from equation (39) in equation (33) we have

$$V(T)_{opt.} = \frac{A\left[A + \mu_0 B\right]}{[A + \mu_0^2 B]}\,\frac{S_y^2}{n} \tag{40}$$

5.2 Optimum Replacement Policy for the Estimator T*

Since, upto the first order of approximations, the expression of $M(T^*)_{opt.}$ given in equation (37) is coinciding with the expression of $V(T)_{opt.}$ given in equation (33). Therefore, the optimum replacement policy of T^* is similar to that of T discussed in Section 5.1.

6. SPECIAL CASE

When the p-auxiliary variates are mutually uncorrelated, i. e., $\rho_{zjzk} = 0,\ \forall j \neq k = 1, 2, ..., p$ then the expression for the optimum value of μ and $V(T)_{opt.}$ reduces to

$$\hat{\mu} = \frac{-A^* \pm \sqrt{A^*(A^* + B^{**})}}{B^{**}} \tag{41}$$

and
$$V(T)_{opt.*} = \frac{A^*\left[A^* + \mu_0 B^{**}\right]}{[A^* + \mu_0^2 B^{**}]}\,\frac{S_y^2}{n} \tag{42}$$

where $A^* = 1 - \sum_{j=1}^{p} \rho_{yzj}^2$

and $B^{**} = 2\,\rho_{yx} \sum_{j=1}^{p} \rho_{yzj}^2 - \rho_{yx}^2\left(1 + \sum_{j=1}^{p} \rho_{yzj}^2\right)$

7. EFFICIENCY COMPARISON

The percent relative efficiencies of T with respect to (i) $\bar{y}_n$, when there is no matching and (ii) $\hat{\hat{\bar{Y}}} = \varphi^*\bar{y}_u + \left(1-\varphi^*\right)\bar{y}'_m$, when no additional auxiliary information was used at any occasion, where $\bar{y}'_m = \bar{y}_m + \beta_{yx}\left(\bar{x}_n - \bar{x}_m\right)$, have been obtained for different choices of the correlations involved. Since, $\bar{y}_n$ and $\hat{\hat{\bar{Y}}}$ are unbiased estimators of $\bar{Y}$, following on the line of Sukhatme *et al.* (1984) the variance of $\bar{y}_n$ and the optimum variance of $\hat{\hat{\bar{Y}}}$ for large N (i.e., $N \to \infty$) are respectively given by

$$V\left(\bar{y}_n\right) = \frac{S_y^2}{n} \tag{43}$$

$$V\left(\hat{\hat{\bar{Y}}}\right)_{opt.*} = \left[1 + \sqrt{\left(1 - \rho_{yx}^2\right)}\right]\frac{S_y^2}{2n} \tag{44}$$

The percent relative efficiencies E_1 and E_2 of T (under optimal condition) with respect to $\bar{y}_n$ and $\hat{\hat{\bar{Y}}}$ respectively are given by

$$E_1 = \frac{V\left(\bar{y}_n\right)}{V\left(T\right)_{opt.*}} \times 100 \quad \text{and} \quad E_2 = \frac{V\left(\hat{\hat{\bar{Y}}}\right)_{opt.*}}{V\left(T\right)_{opt.*}} \times 100$$

7.1 Empirical Study

The expressions of the optimum μ (i.e. μ_0) and the percent relative efficiencies E_1 and E_2 are in terms of population correlation coefficients. Therefore, the values of μ_0, E_1 and E_2 have been computed for different choices of positive correlations. For empirical studies, cases of p = 1, 2 and 3 have been considered.

Case 1. For p = 1, the values of A and B takes the form $A = 1-\rho_{yz}^2$ and $B = 2\rho_{yz}^2\rho_{yx} - \rho_{yx}^2\left(1+\rho_{yz}^2\right)$, which is the work of Singh and Priyanka (2007).

Case 2. For p = 2 and assuming that the two auxiliary variates are correlated i.e., $\rho_{z_1z_2}^2 \neq 0$. The values of A and B are given by

$$A = 1-\rho_{yz1}^2-\rho_{yz2}^2+2\rho_{yz1}\rho_{yz2}\rho_{z1z2} \text{ and}$$

$$B = 2\rho_{yx}\left(\rho_{yz1}^2+\rho_{yz2}^2\right)-\rho_{yx}^2\left(1+\rho_{yz1}^2+\rho_{yz2}^2\right) + 2\left(\rho_{yx}^2-2\rho_{yx}\right)\left\{\rho_{yz1}\rho_{yz2}\rho_{z1z2}\right\}$$

Substituting these values of A and B in equations (39) and (40), we have the values of optimum μ, $V(T)_{opt.^*}$, E_1 and E_2. For different choices of correlations, Tables 1-4 show the optimum values of μ i.e., μ_0 and percent relative efficiencies E_1 and E_2 of T (under optimal condition) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ respectively.

Case 3. For p = 2 and assuming that the two auxiliary variates are uncorrelated i.e., $\rho_{z1z2} = 0$. The values of A^* and B^{**} are given by

$$A^* = 1-\rho_{yz1}^2-\rho_{yz2}^2 \text{ and}$$

$$B^{**} = 2\rho_{yx}\left(\rho_{yz1}^2+\rho_{yz2}^2\right)-\rho_{yx}^2\left(1+\rho_{yz1}^2+\rho_{yz2}^2\right)$$

Hence, using these values in equations (41) and (42), the values of optimum μ, $V(T)_{opt.^*}$, E_1 and E_2 are shown in Table 5.

Case 4. For p = 3 and assuming that the auxiliary variates are correlated i.e., $\rho_{zjzk} \neq 0,\ \forall j \neq k = 1, 2, 3$. In this case the values of A and B takes the following form

Table 1. Optimum values of μ and percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for $\rho_{yx} = 0.3$

ρ_{yz2}		0.4			0.6			0.8		
ρ_{yz1}	ρ_{z1z2}	μ_0	E_1	E_2	μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.3	0.490	138.09	134.90	0.475	166.48	162.65	0.435	248.45	242.72
	0.5	0.497	125.90	123.01	0.488	141.52	138.26	0.466	182.73	178.52
	0.7	0.504	115.74	113.07	0.499	123.20	120.36	0.486	145.13	141.79
	0.9	0.509	107.11	104.65	0.508	109.14	106.63	0.501	120.60	117.82
0.7	0.3	0.467	180.38	176.22	0.447	222.22	217.10	0.386	374.82	366.19
	0.5	0.482	152.96	149.43	0.475	166.48	162.65	0.452	210.36	205.51
	0.7	0.493	132.93	129.87	0.493	133.56	130.48	0.485	148.16	144.75
	0.9	0.502	117.63	114.92	0.506	111.69	109.11	0.504	114.81	112.17
0.9	0.3	0.403	327.52	319.97	0.358	464.87	454.17	*	-	-
	0.5	0.444	227.74	222.50	0.440	237.62	232.15	0.411	304.80	297.78
	0.7	0.469	175.85	171.80	0.477	162.64	158.89	0.473	169.49	165.58
	0.9	0.487	143.66	140.35	0.498	124.26	121.40	0.501	118.61	115.87

Note: * denotes μ_0 does not exist

Table 2. Optimum values of μ and percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for $\rho_{yx} = 0.5$

ρ_{yz2}		0.4			0.6			0.8		
ρ_{yz1}	ρ_{z1z2}	μ_0	E_1	E_2	μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.3	0.506	142.49	132.95	0.485	170.19	158.79	0.436	248.92	232.25
	0.5	0.516	130.52	121.78	0.503	145.86	136.09	0.474	185.92	173.47
	0.7	0.524	120.48	112.41	0.518	127.85	119.29	0.501	149.39	139.39
	0.9	0.532	111.92	104.42	0.530	113.94	106.30	0.520	125.29	116.90
0.7	0.3	0.476	183.65	171.35	0.450	223.88	208.89	0.379	368.28	343.61
	0.5	0.495	157.03	146.52	0.485	170.19	158.79	0.457	212.52	198.28
	0.7	0.509	137.44	128.23	0.509	138.05	128.80	0.498	152.35	142.15
	0.9	0.523	122.35	114.16	0.528	116.46	108.66	0.525	119.56	111.55
0.9	0.3	0.398	323.80	302.11	0.348	452.52	422.20	*	-	-
	0.5	0.447	229.17	213.82	0.441	238.60	222.62	0.408	302.37	282.12
	0.7	0.479	179.27	167.26	0.487	166.46	155.31	0.483	173.11	161.51
	0.9	0.502	147.96	138.05	0.517	128.9	120.27	0.522	123.32	115.06

Note: * denotes μ_0 does not exists

Table 3. Optimum values of μ and percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for $\rho_{ys} = 0.7$

ρ_{yz2}		0.4			0.6			0.8		
ρ_{yz1}	ρ_{z1z2}	μ_0	E_1	E_2	μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.3	0.548	154.30	132.25	0.524	183.77	157.51	0.468	267.55	229.31
	0.5	0.559	141.56	121.32	0.545	157.88	135.32	0.511	200.51	171.85
	0.7	0.569	130.85	112.15	0.562	138.71	118.88	0.542	161.65	138.54
	0.9	0.578	121.73	104.33	0.576	123.88	106.18	0.564	135.98	116.55
0.7	0.3	0.513	198.09	169.78	0.484	240.90	206.47	0.407	394.82	338.39
	0.5	0.535	169.78	145.51	0.524	183.77	157.51	0.492	228.80	196.10
	0.7	0.553	148.92	127.64	0.552	149.57	128.19	0.539	164.80	141.24
	0.9	0.567	132.86	113.87	0.573	126.58	108.48	0.570	129.88	111.31
0.9	0.3	0.427	347.34	297.70	0.373	484.87	415.57	*	-	-
	0.5	0.481	246.52	211.28	0.474	256.56	219.89	0.438	324.49	278.11
	0.7	0.517	193.43	165.79	0.526	179.80	154.10	0.521	186.88	160.16
	0.9	0.543	160.12	137.23	0.561	139.83	119.85	0.566	133.88	114.75

Note: * denotes μ_0 does not exist

Table 4. Optimum values of μ and percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for $\rho_{ys} = 0.9$

ρ_{yz2}		0.4			0.6			0.8		
ρ_{yz1}	ρ_{z1z2}	μ_0	E_1	E_2	μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.3	0.661	186.14	133.64	0.637	223.37	160.37	0.580	331.44	237.96
	0.5	0.672	170.17	122.17	0.658	190.64	136.87	0.624	244.72	175.70
	0.7	0.682	156.84	112.60	0.675	166.62	119.62	0.655	195.37	140.27
	0.9	0.691	145.53	104.48	0.689	148.19	106.39	0.677	163.22	117.18
0.7	0.3	0.626	241.63	173.48	0.597	296.74	213.05	0.515	500.42	359.28
	0.5	0.648	205.64	147.64	0.636	223.38	160.37	0.604	281.09	201.81
	0.7	0.666	179.39	128.79	0.665	180.20	129.37	0.652	199.35	143.12
	0.9	0.680	159.33	114.39	0.686	151.53	108.79	0.683	155.63	111.73
0.9	0.3	0.537	436.81	313.61	0.479	622.66	447.03	*	-	-
	0.5	0.593	304.04	218.28	0.587	317.10	227.66	0.549	406.42	291.79
	0.7	0.629	235.68	169.21	0.640	218.34	156.75	0.634	227.33	163.21
	0.9	0.656	193.45	138.89	0.674	168.02	120.63	0.680	160.60	115.31

Note: * denotes μ_0 does not exist.

Table 5. Optimum values of μ and Percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$

ρ_{yx}		0.3			0.7			0.9		
ρ_{yz2}	ρ_{yz1}	μ_0	E_1	E_2	μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.4	0.477	161.71	157.98	0.528	178.34	153.28	0.641	217.11	155.87
	0.6	0.444	227.74	222.50	0.481	246.52	211.28	0.593	304.04	218.28
	0.8	0.325	591.16	577.55	0.336	610.76	523.47	0.438	796.37	571.74
	0.9	*	-	-	*	-	-	*	-	-
0.7	0.3	0.435	248.45	242.73	0.468	267.55	229.31	0.580	331.44	237.96
	0.5	0.355	473.85	462.94	0.370	493.83	423.25	0.476	634.92	455.84
	0.7	*	-	-	*	-	-	*	-	-
	0.9	*	-	-	*	-	-	*	-	-
0.9	0.3	0.208	1387.00	1355.00	0.210	1402.40	1202.00	0.289	1931.50	1386.70
	0.5	*	-	-	*	-	-	*	-	-
	0.7	*	-	-	*	-	-	*	-	-
	0.9	*	-	-	*	-	-	*	-	-

Note: * denotes μ_0 does not exist.

$$A = 1-\rho^2_{yz1} - \rho^2_{yz2} - \rho^2_{yz3} + 2\left[\rho_{yz1}\rho_{yz2}\rho_{z1z2} + \rho_{yz1}\rho_{yz3}\rho_{z1z3} + \rho_{yz2}\rho_{yz3}\rho_{z2z3}\right]$$

$$B = 2\rho_{yx}\left(\rho^2_{yz1} + \rho^2_{yz2} + \rho^2_{yz3}\right) - \rho^2_{yx}\left(1 + \rho^2_{yz1} + \rho^2_{yz2} + \rho^2_{yz3}\right) + 2\left(\rho^2_{yx} - 2\rho_{yx}\right)\left[\rho_{yz1}\rho_{yz2}\rho_{z1z2} + \rho_{yz1}\rho_{yz3}\rho_{z1z3} + \rho_{yz2}\rho_{yz3}\rho_{z2z3}\right]$$

In this case there are seven different correlations. For few sets of these seven correlations optimum values of μ i.e., μ_0 and percent relative efficiencies E_1 and E_2 of T (under optimal condition) with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ respectively have been computed and shown below:

Set 1: $\rho_{yx} = 0.3$, $\rho_{z1z2} = 0.3$, $\rho_{z1z3} = 0.6$, $\rho_{z2z3} = 0.4$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.6$, $\rho_{yz3} = 0.7$, $\mu_0 = 0.5010$, $E_1 = 119.85$, $E_2 = 117.09$

Set 2: $\rho_{yx} = 0.3$, $\rho_{z1z2} = 0.3$, $\rho_{z1z3} = 0.6$, $\rho_{z2z3} = 0.4$, $\rho_{yz1} = 0.9$, $\rho_{yz2} = 0.6$, $\rho_{yz3} = 0.7$, $\mu_0 = 0.4944$, $E_1 = 130.80$, $E_2 = 127.79$

Set 3: $\rho_{yx} = 0.3$, $\rho_{z1z2} = 0.3$, $\rho_{z1z3} = 0.6$, $\rho_{z2z3} = 0.4$, $\rho_{yz1} = 0.9$, $\rho_{yz2} = 0.8$, $\rho_{yz3} = 0.7$, $\mu_0 = 0.4888$, $E_1 = 140.47$, $E_2 = 137.23$

Set 4: $\rho_{yx} = 0.3$, $\rho_{z1z2} = 0.3$, $\rho_{z1z3} = 0.6$, $\rho_{z2z3} = 0.4$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.8$, $\rho_{yz3} = 0.9$, $\mu_0 = 0.4847$, $E_1 = 147.77$, $E_2 = 144.37$

Case 5: For p = 3 and assuming the auxiliary variates are independent (uncorrelated) i.e., $\rho_{zhk} = 0$, $\forall_j \neq k = 1, 2, 3$ In this case the values of A and B takes the following form

$$A = 1 - \rho^2_{yz1} - \rho^2_{yz2} - \rho^2_{yz3}$$

$$B = 2\rho_{yx}\left(\rho^2_{yz1} + \rho^2_{yz2} + \rho^2_{yz3}\right) - \rho^2_{yx}\left(1 + \rho^2_{yz1} + \rho^2_{yz2} + \rho^2_{yz3}\right)$$

For few sets of above four correlations, the values of μ_0, E_1 and E_2 are shown below.

Set 1: $\rho_{yx} = 0.3$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.6$, $\rho_{yz3} = 0.5$
$\mu_0 = 0.3487$, $E_1 = 498.08$, $E_2 = 486.61$

Set 2: $\rho_{yx} = 0.5$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.6$, $\rho_{yz3} = 0.5$
$\mu_0 = 0.3384$, $E_1 = 483.47$, $E_2 = 451.08$

Set 3: $\rho_{yx} = 0.7$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.6$, $\rho_{yz3} = 0.5$
$\mu_0 = 0.3626$, $E_1 = 518.00$, $E_2 = 443.97$

Set 4: $\rho_{yx} = 0.9$, $\rho_{yz1} = 0.5$, $\rho_{yz2} = 0.8$, $\rho_{yz3} = 0.9$
$\mu_0 = 0.4677$, $E_1 = 668.09$, $E_2 = 479.65$

8. CONCLUSION

The following conclusions can be read-out from the present study.

1. From Tables 1-4, it is vindicated that for fixed values of ρ_{yx}, ρ_{z1z2} and ρ_{yz1}, the optimum values of μ decreases and E_1 and E_2 increases with increasing values of ρ_{yz2}. Similarly, for fixed values of ρ_{yx}, ρ_{z1z2} and ρ_{yz2}, the optimum values of μ decreases and E_1 and E_2 increases with increasing values of ρ_{yz1}. These patterns indicate that smaller fresh sample at current occasion is required, if highly correlated auxiliary variates are available.

2. For fixed values of ρ_{z1z2}, ρ_{yz1} and ρ_{yz2}, the optimum values of μ and E_1 increases with increasing values of ρ_{yx} while decreasing pattern in E_2 is observed. This behavior is in agreement with Sukhatme *et al.* (1984) results, which explains that more the value of ρ_{yx} more the fraction of fresh sample is required at current occasion.

3. For fixed values of ρ_{yx}, ρ_{yz1} and ρ_{yz2} the optimum values of μ increases with increasing values of ρ_{z1z2}, while E_1 and E_2 are decreasing with increasing trends in ρ_{z1z2}, it means that auxiliary variates are quite sensitive with respect to the relation between them.

4. From Table 5 i.e., when auxiliary variates are uncorrelated, it has been observed that for fixed values of $\rho_{yz1}$and ρ_{yz2} the optimum values of μ and E_1 increases with the increasing values of ρ_{yx}, while no definite patterns are observed in E_2.

5. For fixed values of ρ_{yz1} and ρ_{yx} the optimum values of μ decreases, while E_1 and E_2 increases abruptly with the increasing values of ρ_{yz2}. Similar patterns are visible for the case when the values of ρ_{yz2} and ρ_{yx} are fixed and increasing values of ρ_{yz1} are observed.

6. For p = 3 and when the three auxiliary variates are mutually correlated then for fixed values of ρ_{yx}, ρ_{z1z3}, ρ_{z2z3}, ρ_{z1z2}, ρ_{yz2} and ρ_{yz3} the values of optimum μ decrease, while E_1 and E_2 increase with the increasing values of ρ_{yz1}. Similar patterns are observed if the case for the increasing values of ρ_{yz2} or ρ_{yz3} is taken into account.
7. For p = 3 and when the three auxiliary variates are mutually independent, we observed that no specific pattern is seen and for so many combinations of correlations the optimum values of μ do not exist. Hence, it is clear that the auxiliary variates are sensitive with respect to the relation among themselves.
8. The results obtained for p = 1 and p = 2 are quite appreciable, while when the number of auxiliary variates increases, we do not get an encouraging results. Therefore, it is wise to use utmost two auxiliary variates out of several available auxiliary variates. The two auxiliary variates may be chosen, which are highly correlated with the study variate.

Thus, it is clear that the use of the auxiliary variates are highly rewarding in terms of the proposed estimators. It is also clear that if highly correlated auxiliary variates are used, relatively only a smaller fraction of sample on the second (current) occasion is desired to be replaced by a fresh sample reducing cost of the survey. Hence, they can be recommended for further use.

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